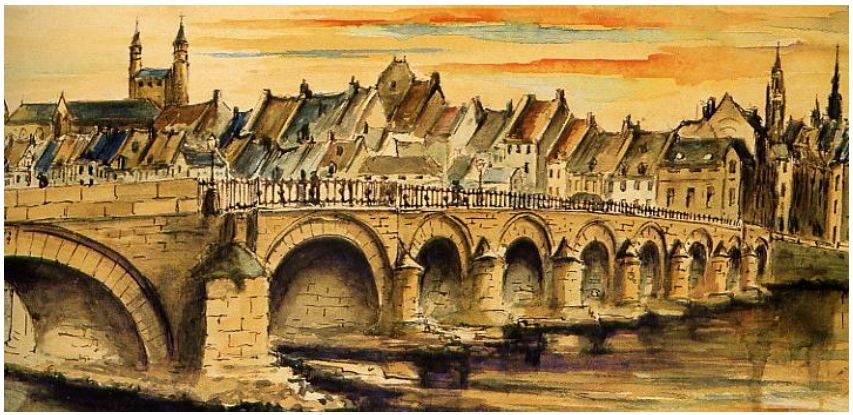
Evolutionary Games and Local Interactions



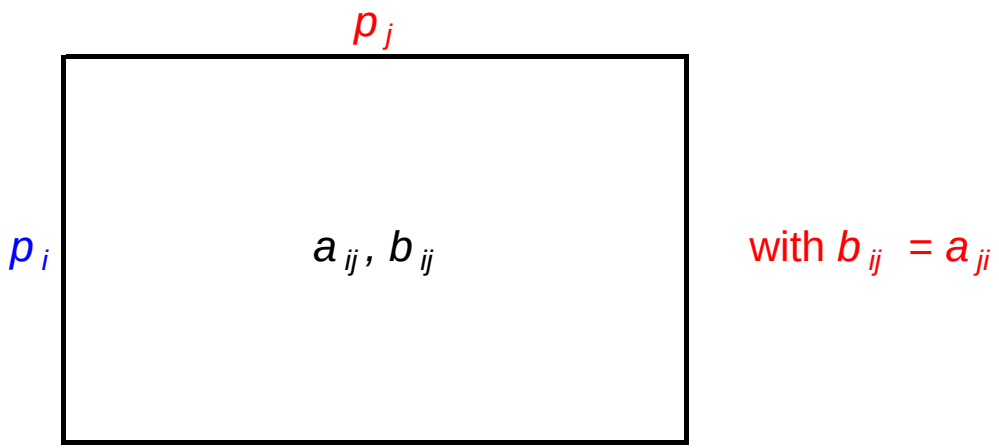
Frank Thuijsman

joint work with Philippe Uyttendaele, Mandy Tak, Ronald Westra

Outline

1973, John Maynard Smith and George Price

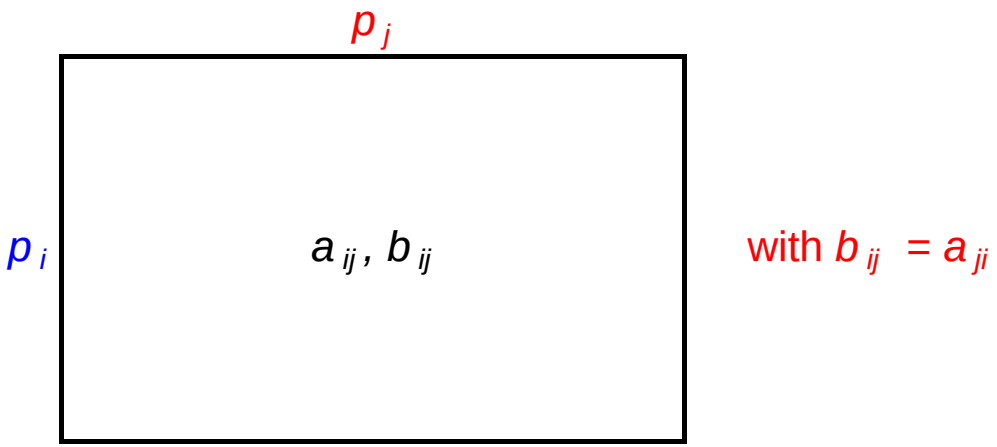
Evolutionary Games



- Population of different types playing against itself.
- Population distribution $p = (p_1, p_2, \ldots, p_n)$.
- Type k has fitness $e_k A p^{\mathrm{T}}$ in population p.
- Concept of evolutionarily stable strategies (ESS).

The ESS Concept

Evolutionary Games

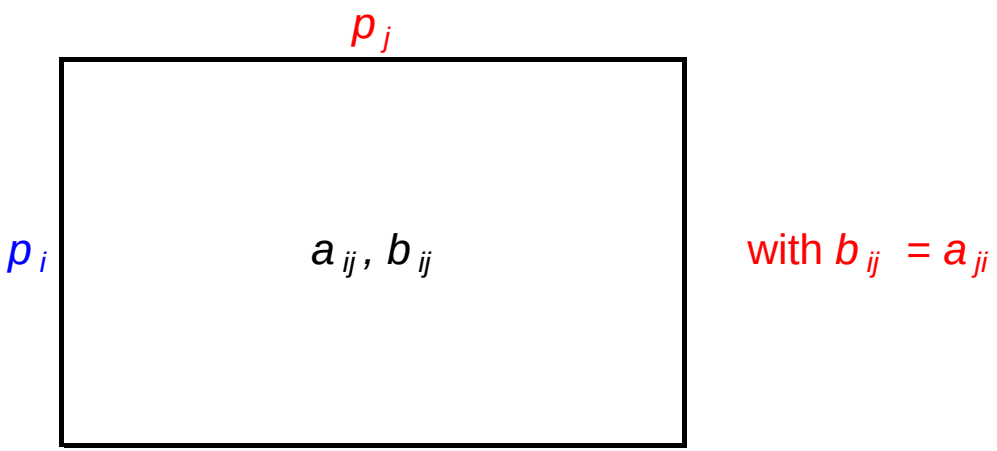


ESS: Population distribution $p = (p_1, p_2, \ldots, p_n)$ with

- $pAp^{\mathrm{T}} \geq qAp^{\mathrm{T}}$ for all q
- If $q \neq p$ and $qAp^{\mathrm{T}} = pAp^{\mathrm{T}}$, then $pAq^{\mathrm{T}} > qAq^{\mathrm{T}}$

The Replicator Dynamic by Taylor and Jonker, 1978

Evolutionary Games



Population development by the replicator equation:

- $\dot{p}_k = p_k \left(e_k A p^{\mathrm{T}} - p A p^{\mathrm{T}} \right)$

Remarks on ESS and Replicator Dynamic

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- Model assumes a homogeneously mixed population

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- What if the population is not homogeneously mixed?

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- Replicator dynamic based on average payoffs

Remarks on ESS and Replicator Dynamic

- Model assumes a homogeneously mixed population
- What if the population is not homogeneously mixed?
- Replicator dynamic based on average payoffs
- What if the process is driven by actual payoffs?

Several Models of Local Interactions

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- Basic model: playing with the neighbours

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- Varying fitness in space and time

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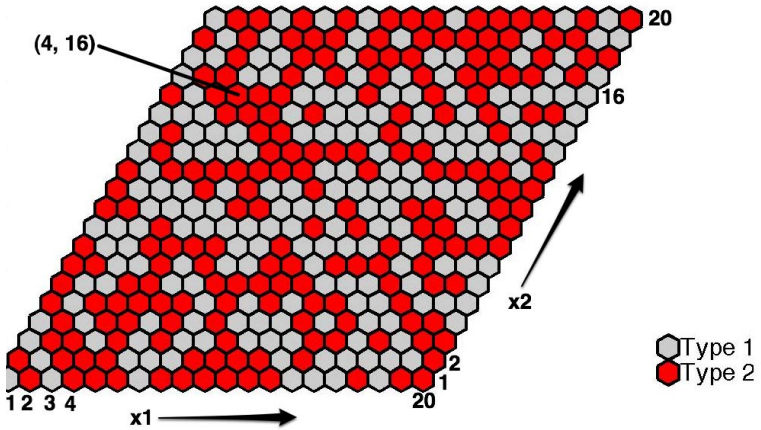
- Basic model: playing with the neighbours
- Varying fitness in space and time
- Actual payoffs versus average payoffs

Several Models of Local Interactions

- Basic model: playing with the neighbours
- Varying fitness in space and time
- Actual payoffs versus average payoffs
- Different types of neighbourhoods

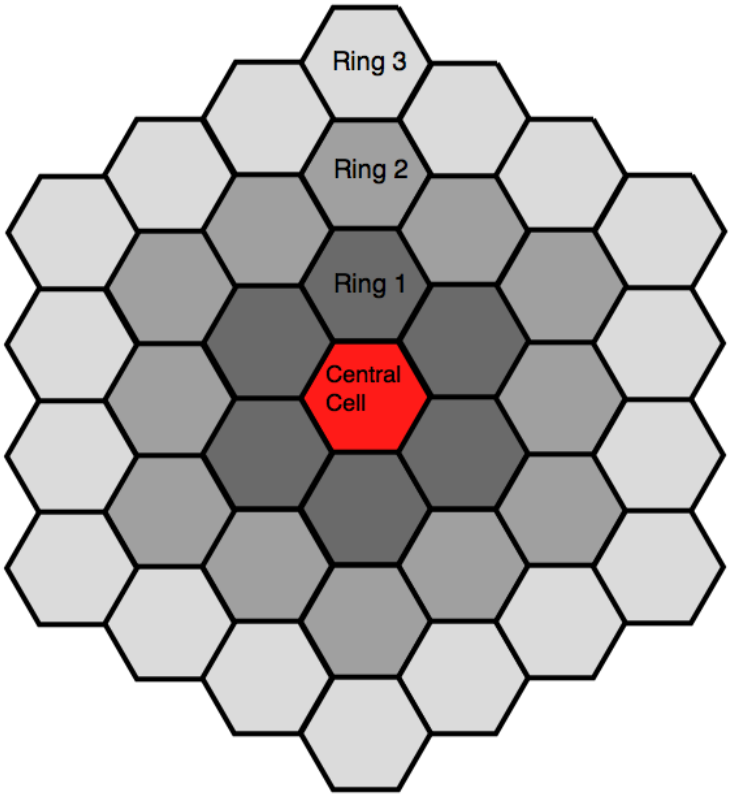
Basic Model: Playing with the Neighbours

- Field consists of hexagonal cells.
- Each cell contains exactly one type.



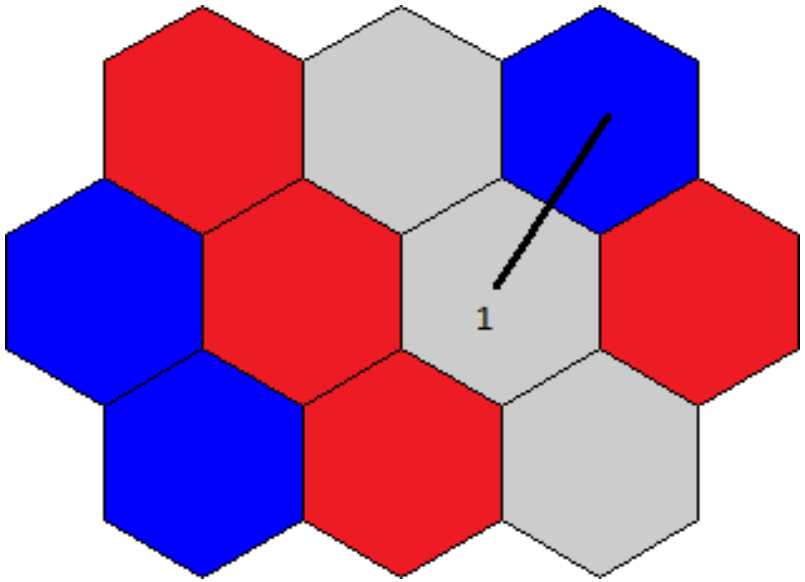
Basic Model - Neighbourhood

- Each cell *c* interacts with its neighbourhood.
- Neighbourhood is defined by distance *M*: all cells within distance *M* belong to neighbourhood of *c*.



Updating the Field

$$\begin{array}{c} \\ \blacksquare \\ \blacksquare \\ \blacksquare \end{array} \begin{array}{c} \begin{array}{ccc} \blacksquare & \blacksquare & \blacksquare \end{array} \\ \begin{pmatrix} 2 & 3 & 1 \\ 0 & 5 & 2 \\ 4 & -1 & 1 \end{pmatrix} \end{array}$$



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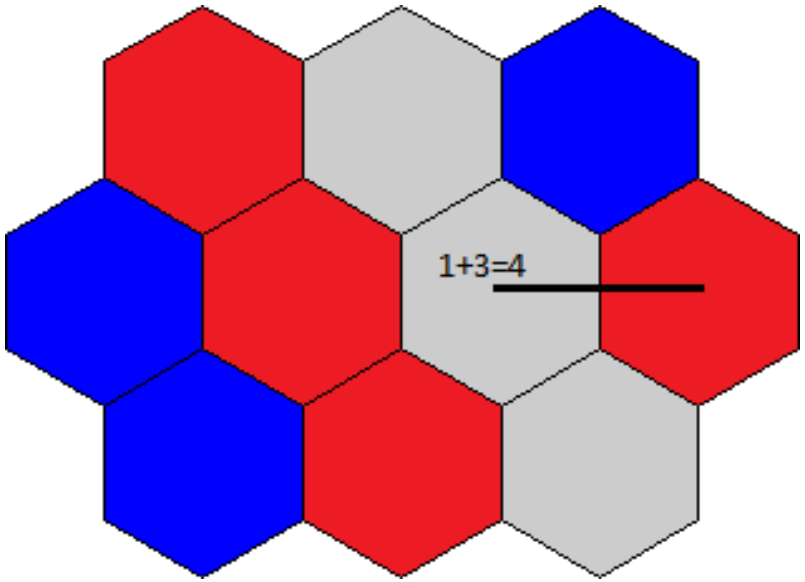

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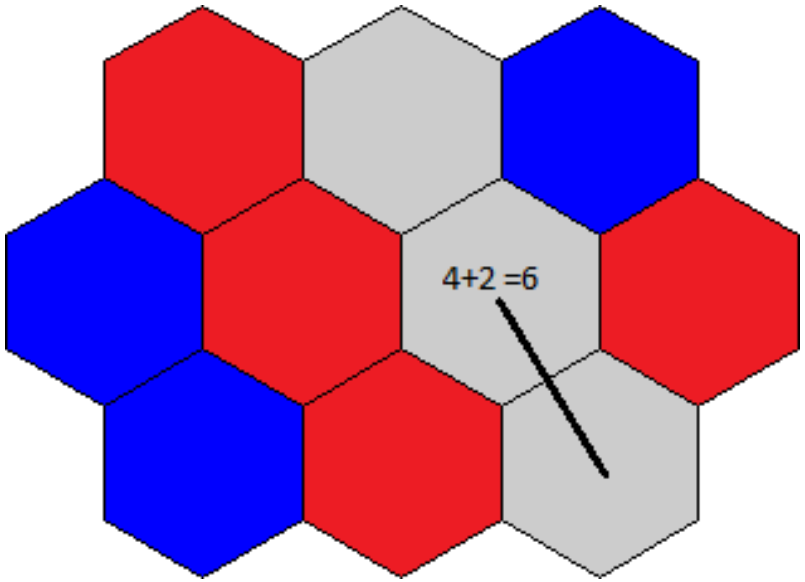

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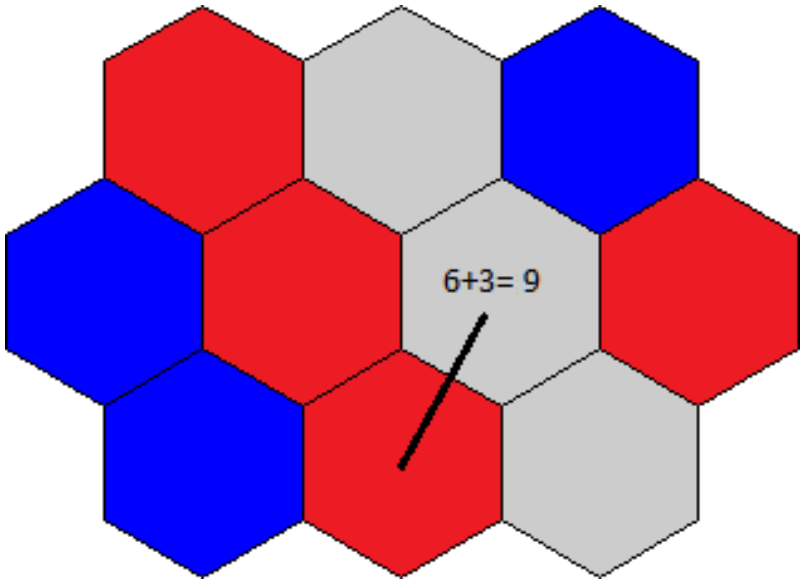

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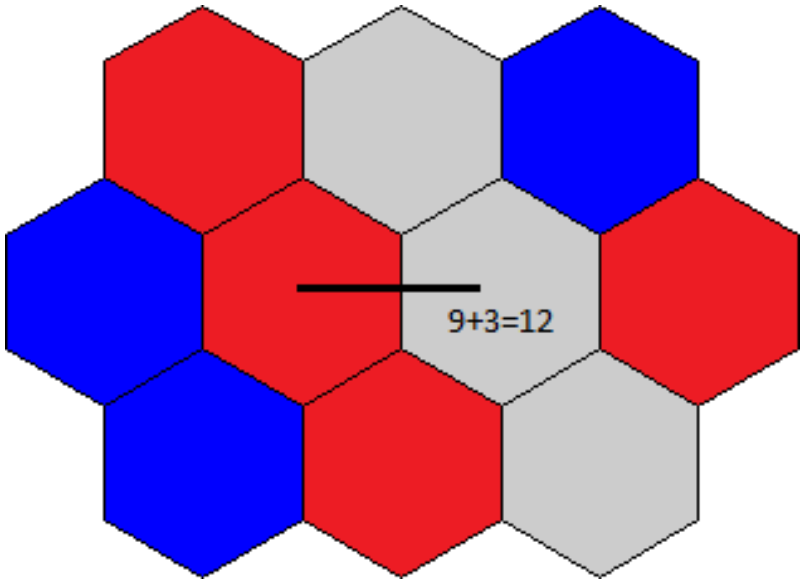

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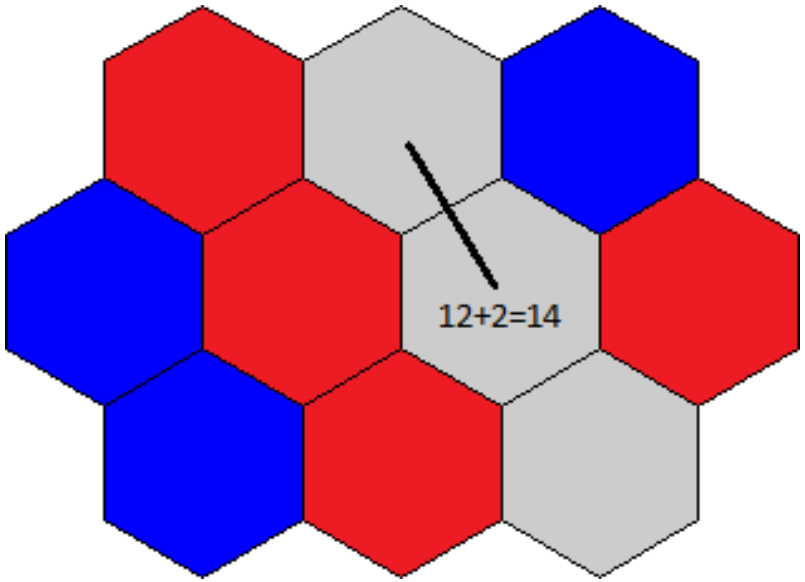

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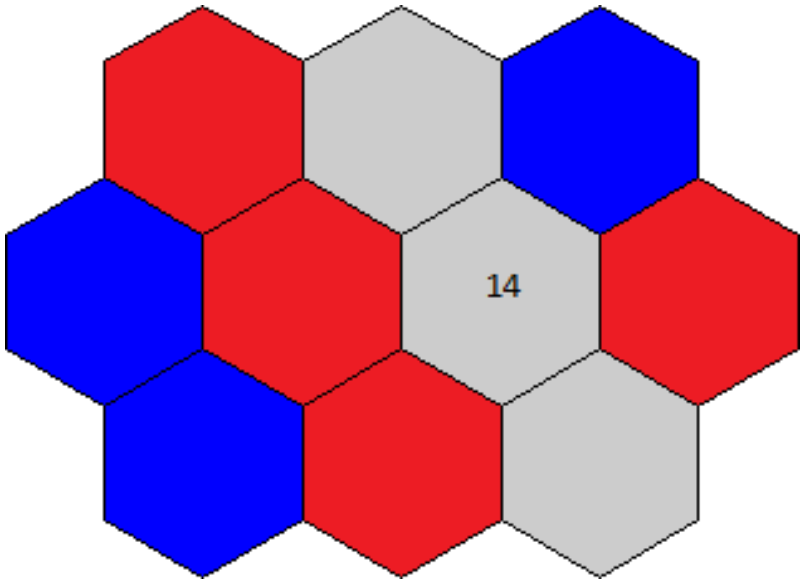

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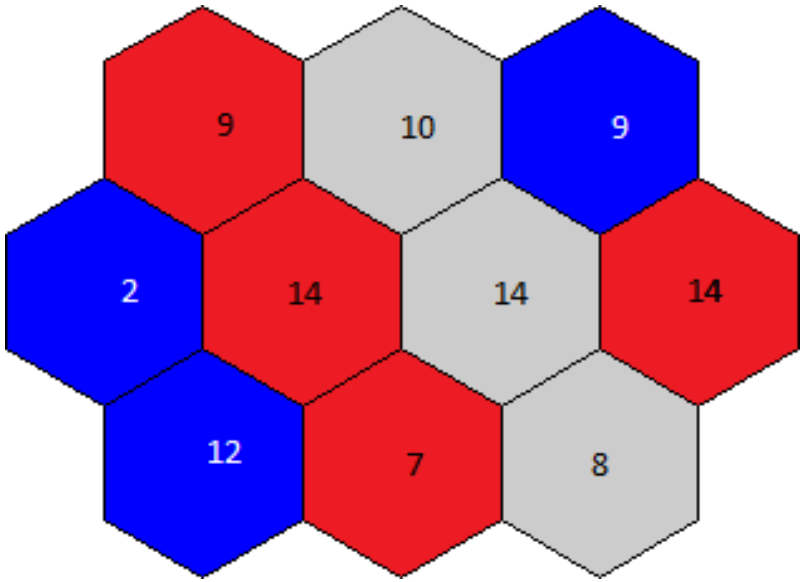
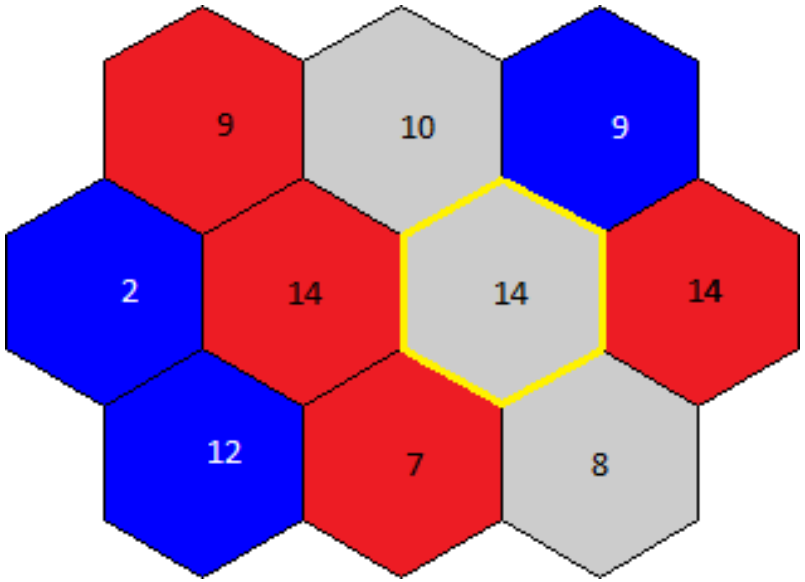


Updating the Field

	total
	32
	35
	9



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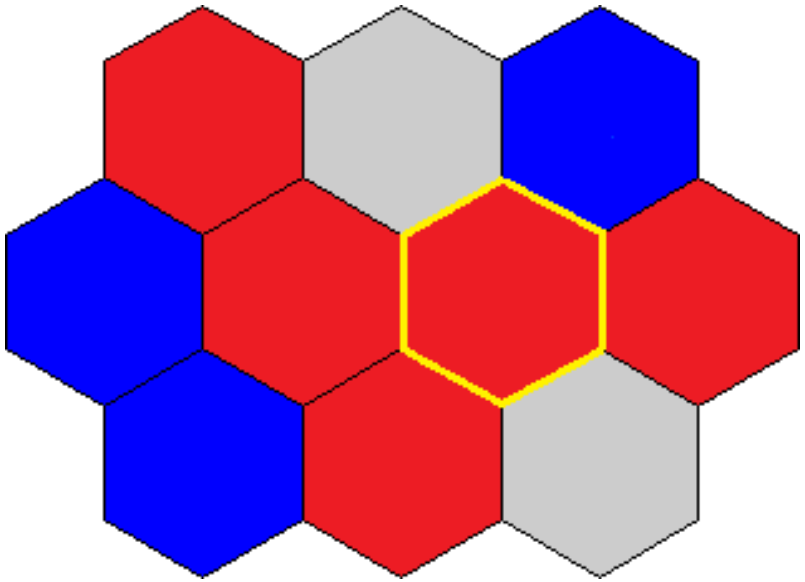
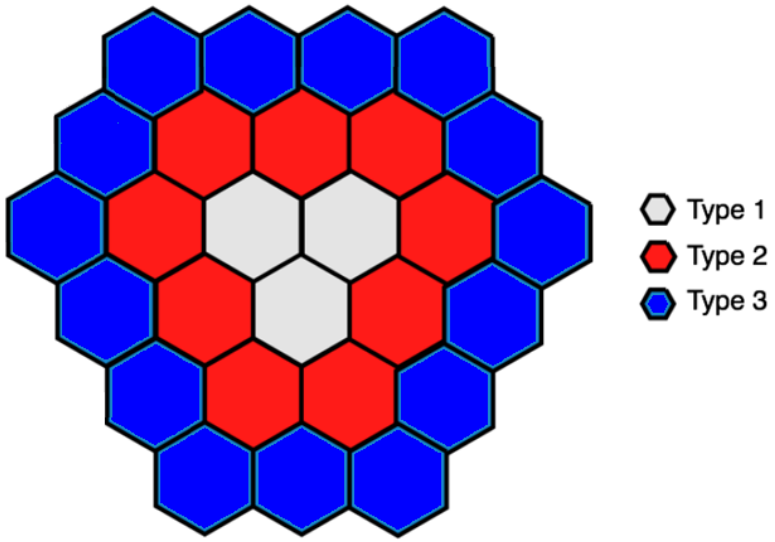


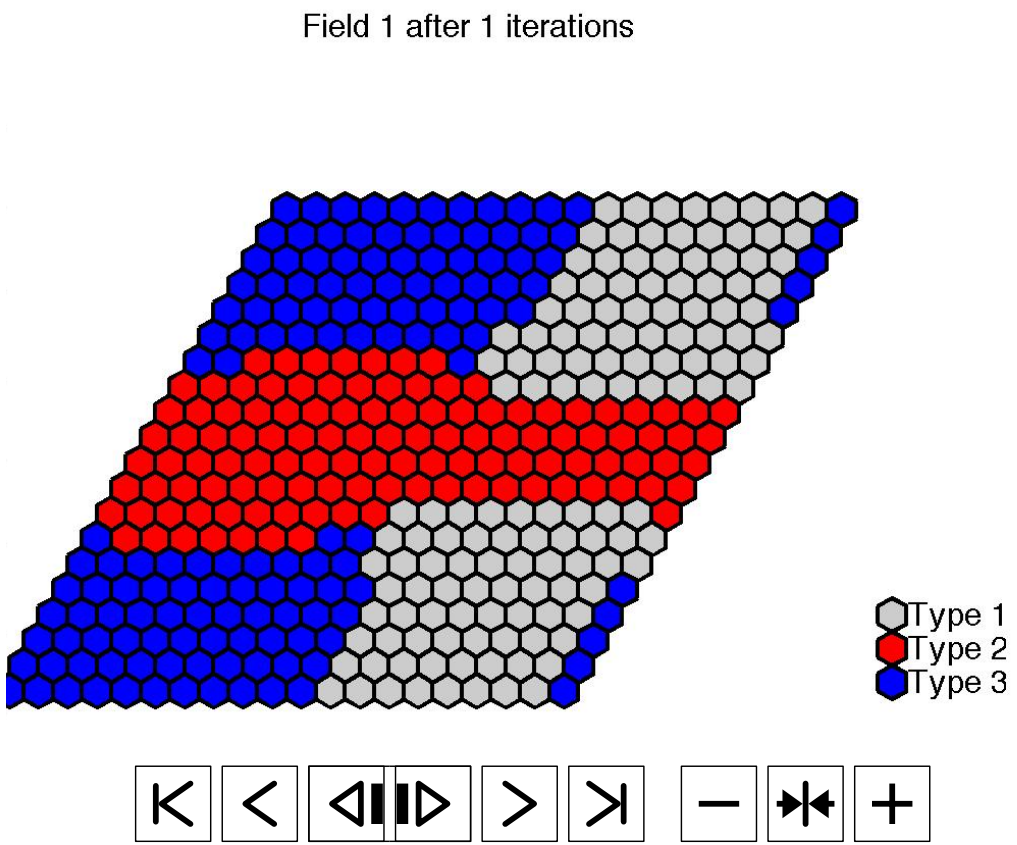
Local Stability

$$\begin{array}{c} \\ \blacksquare \\ \blacksquare \\ \blacksquare \end{array} \begin{array}{c} \begin{array}{ccc} \blacksquare & \blacksquare & \blacksquare \end{array} \\ \begin{pmatrix} 0 & 6 & 0 \\ 12 & 0 & 0 \\ 13 & 7 & 1 \end{pmatrix} \end{array}$$



Dynamic Stability: Predator-Prey Behaviour

$$\begin{array}{c} \\ \blacksquare \\ \blacksquare \\ \blacksquare \end{array} \begin{array}{c} \begin{array}{ccc} \blacksquare & \blacksquare & \blacksquare \end{array} \\ \begin{pmatrix} 1 & 8 & 1 \\ 1 & 1 & 8 \\ 8 & 1 & 1 \end{pmatrix} \end{array}$$



Varying Fitness in Space and Time

$$A(x_2,t) = \begin{pmatrix} 0 & 0 & 2+\sin\left(\frac{t+x_2}{20}\cdot 2\pi\right) \\ 0.7 & 0 & 0.7 \\ 2-\sin\left(\frac{t+x_2}{20}\cdot 2\pi\right) & 0 & 0 \end{pmatrix}$$

Varying Fitness in Space and Time

This example is dedicated to Sylvain

$$A(x_2,t) = \begin{pmatrix} 0 & 0 & 2+\sin\left(\frac{t+x_2}{20}\cdot 2\pi\right) \\ 0.7 & 0 & 0.7 \\ 2-\sin\left(\frac{t+x_2}{20}\cdot 2\pi\right) & 0 & 0 \end{pmatrix}$$

Varying Fitness in Space and Time

We All Need Love!

$$A(x_2,t) = \begin{pmatrix} 0 & 0 & 2+\sin\left(\frac{t+x_2}{20}\cdot 2\pi\right) \\ 0.7 & 0 & 0.7 \\ 2-\sin\left(\frac{t+x_2}{20}\cdot 2\pi\right) & 0 & 0 \end{pmatrix}$$

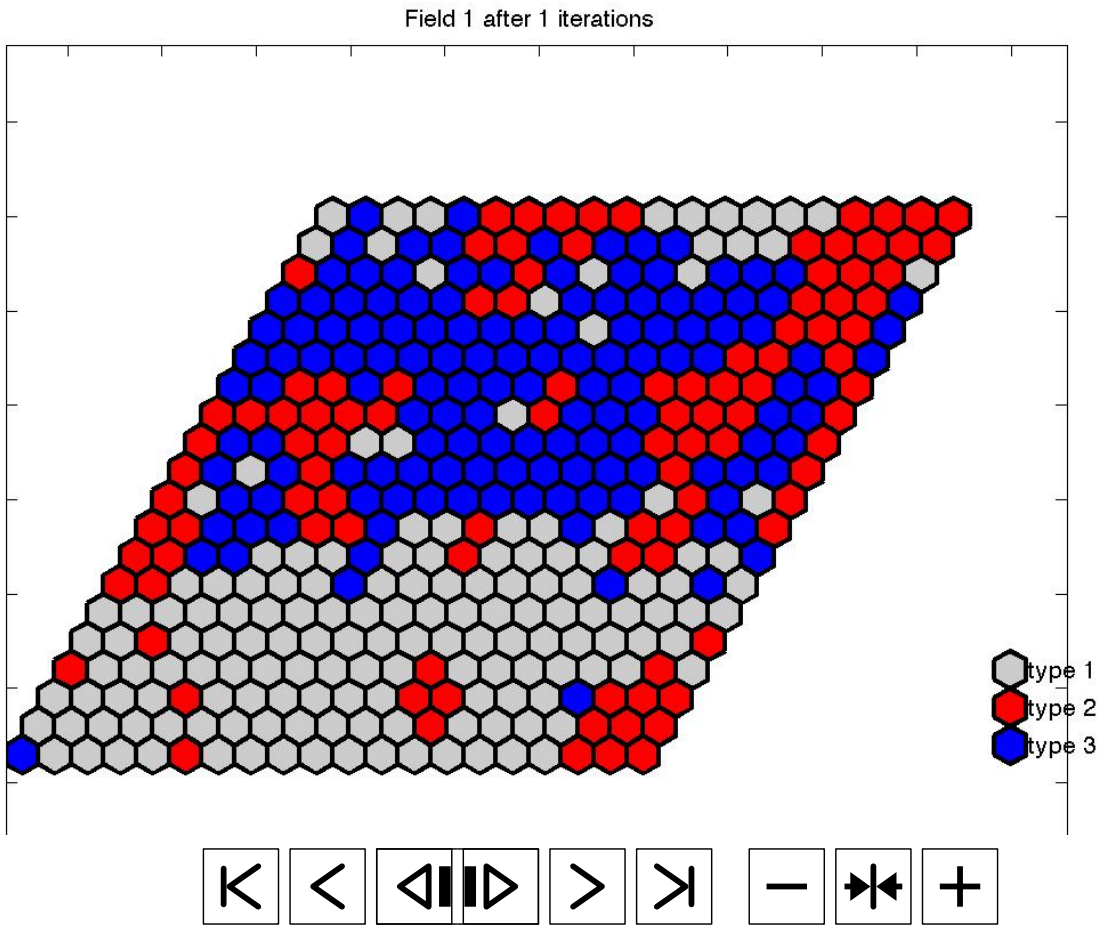
Varying Fitness in Space and Time

We All Need Sylvain!

$$A(x_2,t) = \begin{pmatrix} 0 & 0 & 2+\sin\left(\frac{t+x_2}{20}\cdot 2\pi\right) \\ 0.7 & 0 & 0.7 \\ 2-\sin\left(\frac{t+x_2}{20}\cdot 2\pi\right) & 0 & 0 \end{pmatrix}$$

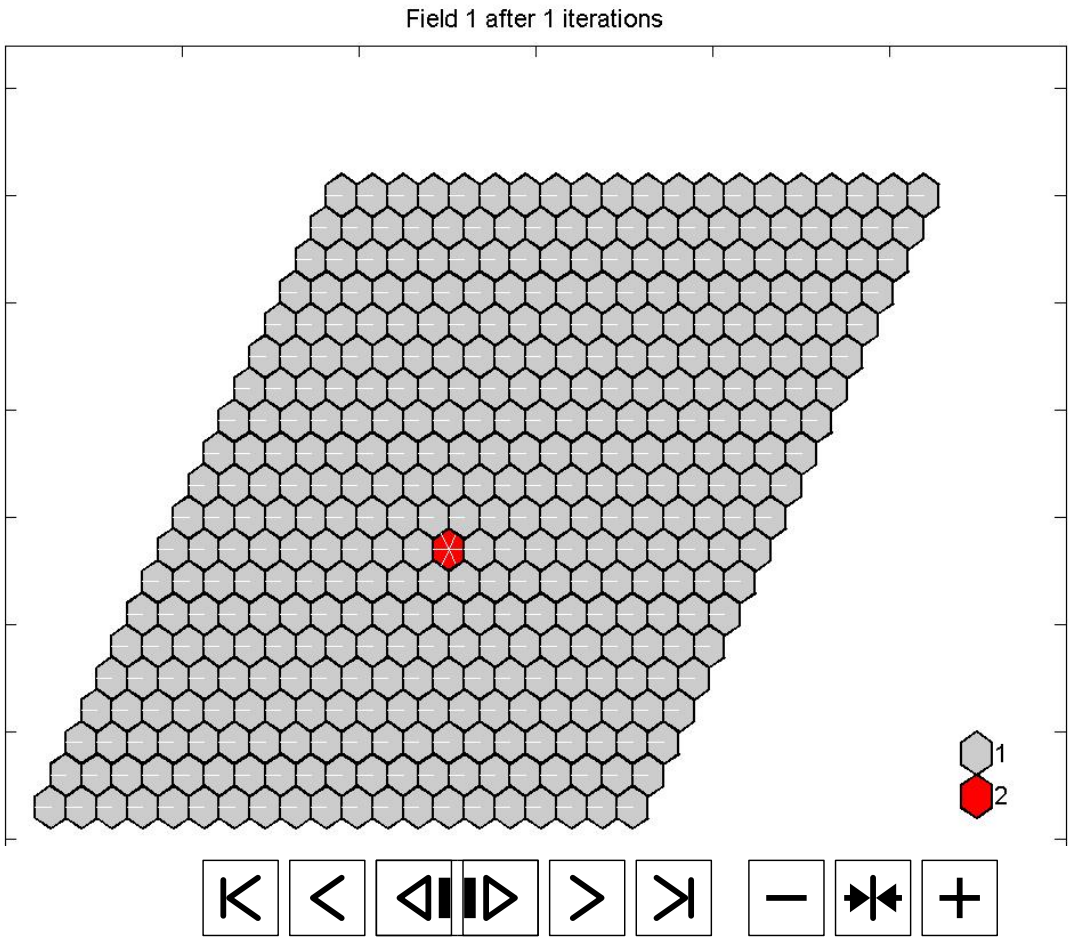
Dynamic Stability

Happy Birthday Sylvain!



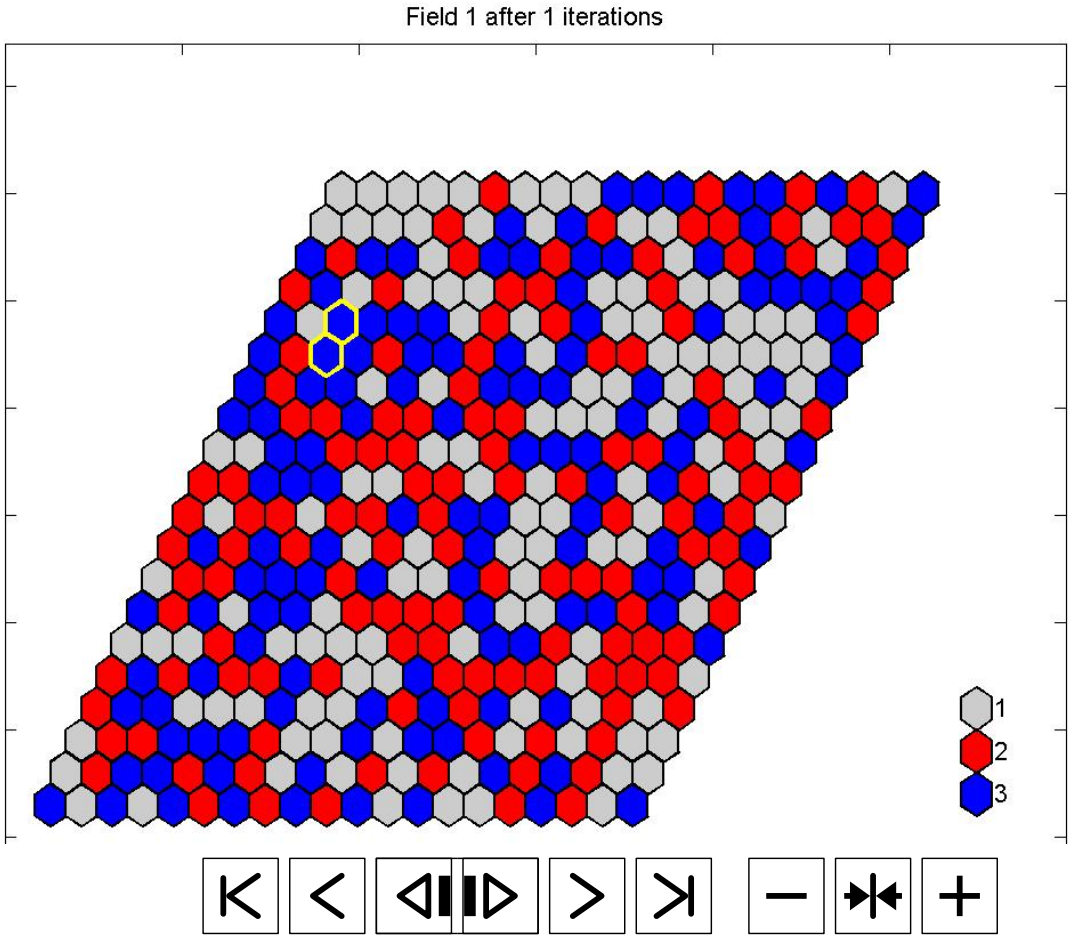
Directional Interactions – Using Only One Neighbour

$$\begin{pmatrix} 0 & 0 \\ 2 & 1 \end{pmatrix}$$



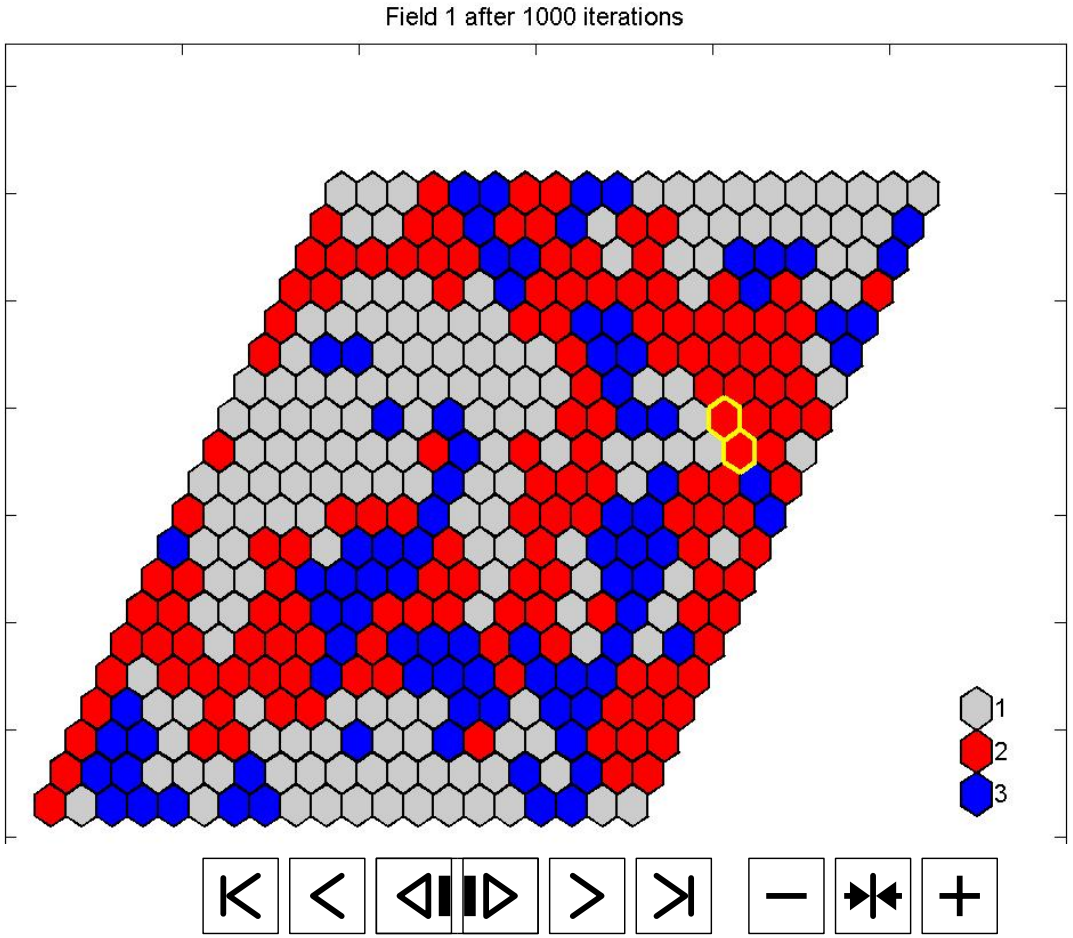
Local Updating without Neighbour Support – Small Time Steps

$$\begin{array}{c} \\ \blacksquare \\ \blacksquare \\ \blacksquare \end{array} \begin{array}{c} \begin{array}{ccc} \blacksquare & \blacksquare & \blacksquare \end{array} \\ \begin{pmatrix} 0 & 1 & 0 \\ 3 & 0 & 0 \\ -2 & 2 & 0 \end{pmatrix} \end{array}$$



Local Updating without Neighbour Support – Big Time Steps

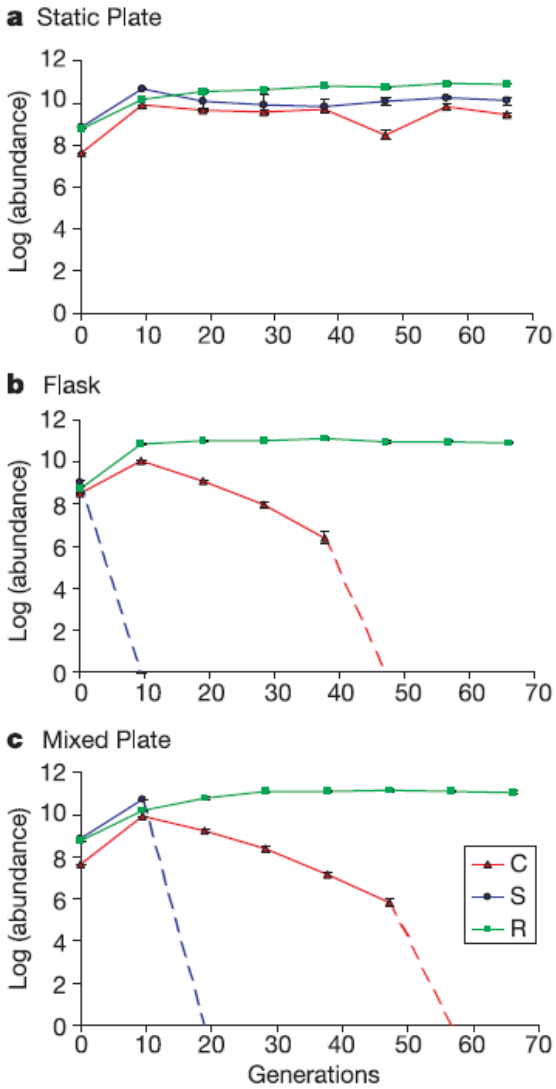
$$\begin{array}{c} \\ \blacksquare \\ \blacksquare \\ \blacksquare \end{array} \begin{array}{c} \begin{array}{ccc} \blacksquare & \blacksquare & \blacksquare \end{array} \\ \begin{pmatrix} 0 & 1 & 0 \\ 3 & 0 & 0 \\ -2 & 2 & 0 \end{pmatrix} \end{array}$$

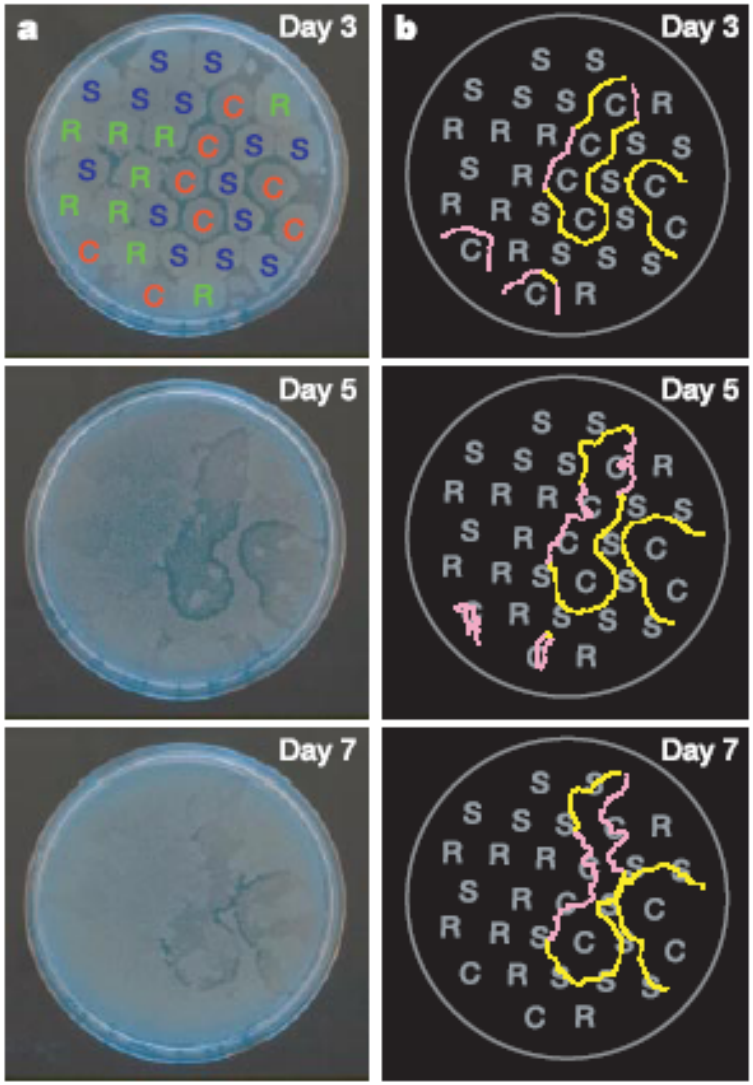


Applications in Micro-Biology

- B. Kerr, M.A. Riley, M.W. Feldman, B.J.M. Bohannan (2002): Local dispersal promotes biodiversity in a real-life game of rock-paper-scissors *Nature* 418, 171-174.
- B.C. Kirkup, M.A. Riley (2004): Antibiotic-mediated antagonism leads to a bacterial game of rock-paper-scissors in vivo. *Nature* 428, 412-414.

Applications in Micro-Biology





Applications in Micro-Biology

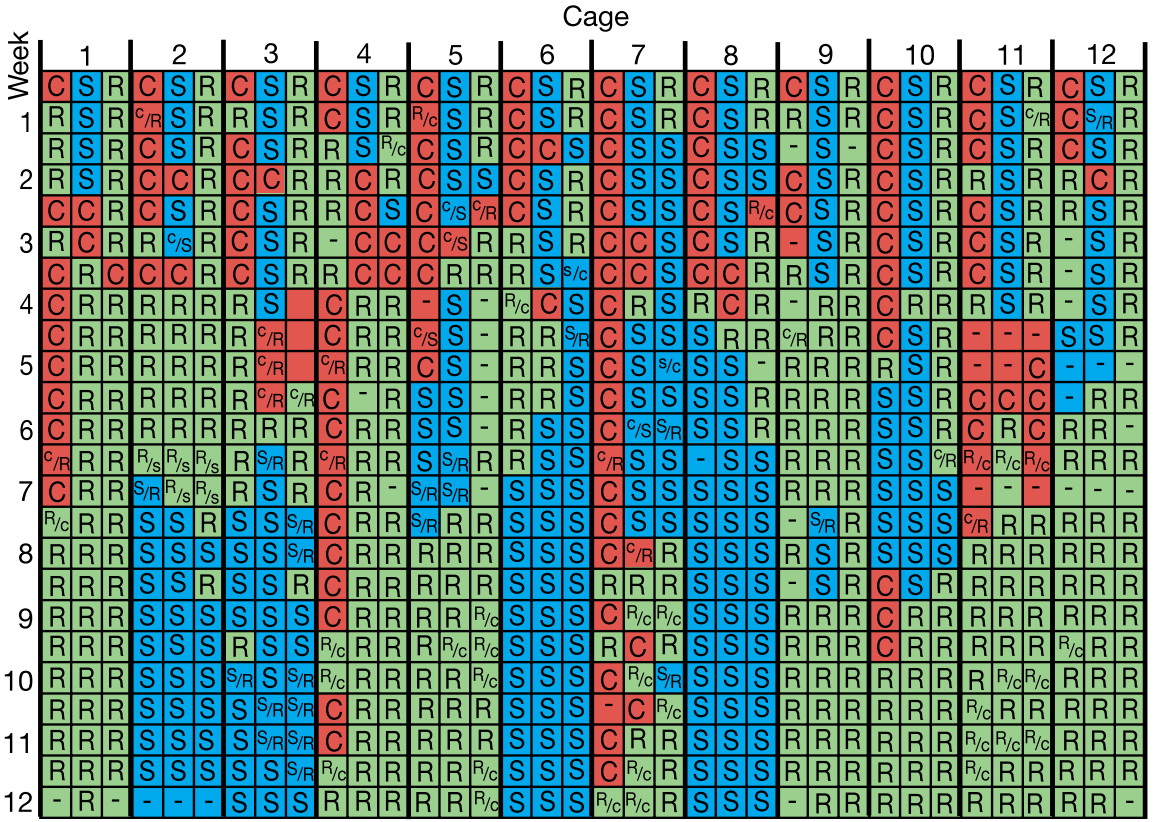


Figure 1 Occupation of co-caged mice by dominant strain: colicin E1. The boxes are coloured to show the dominant strain occupying each mouse. Red represents colicinogenic (C_{E1}); green, resistant; blue, sensitive.

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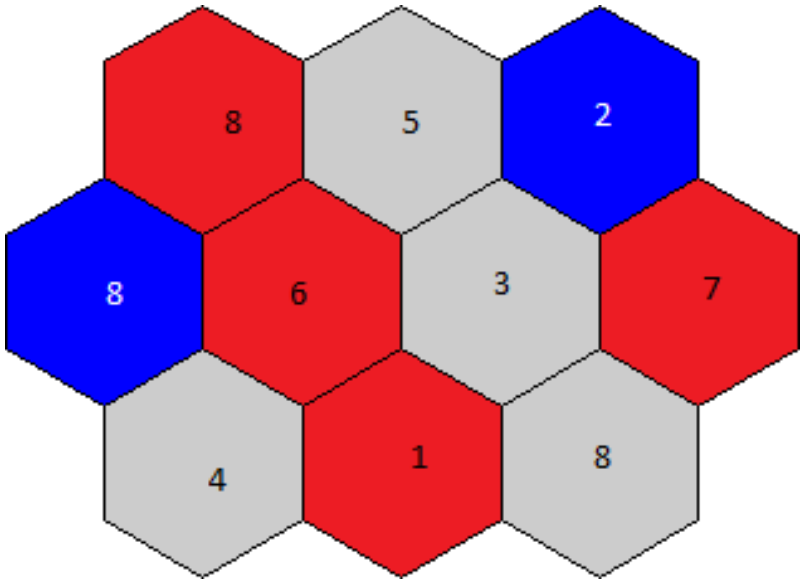
Numerical Interactions with Neighbour Support

$$\begin{array}{c} \\ \blacksquare \\ \blacksquare \\ \blacksquare \end{array} \begin{array}{c} \begin{array}{ccc} \blacksquare & \blacksquare & \blacksquare \end{array} \\ \begin{pmatrix} 2 & 3 & 1 \\ 0 & 5 & 2 \\ 4 & -1 & 1 \end{pmatrix} \end{array}$$



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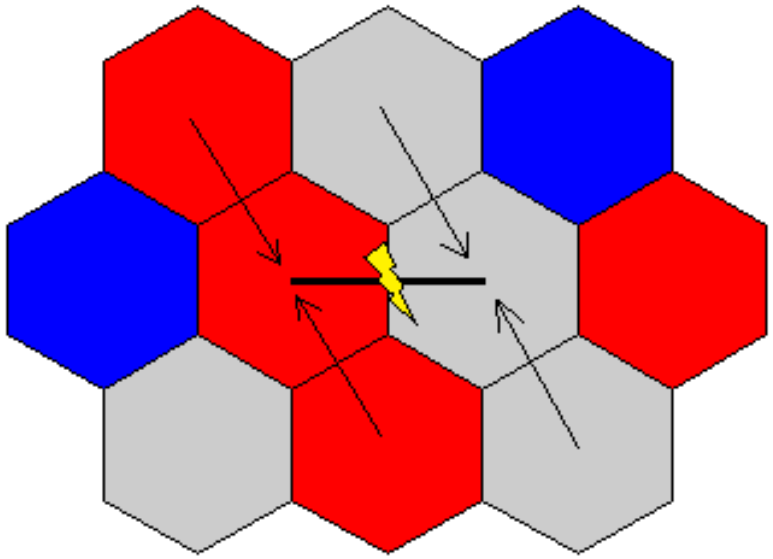

Numerical Interactions with Neighbour Support

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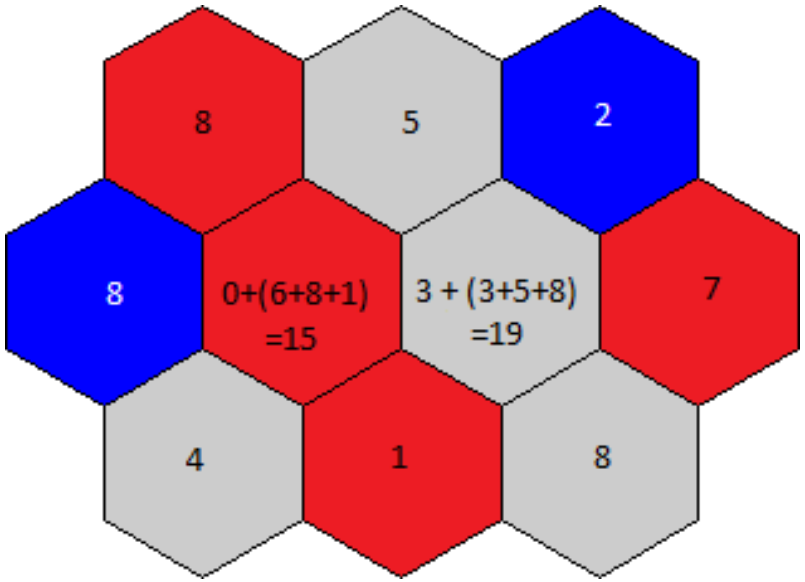

Numerical Interactions with Neighbour Support

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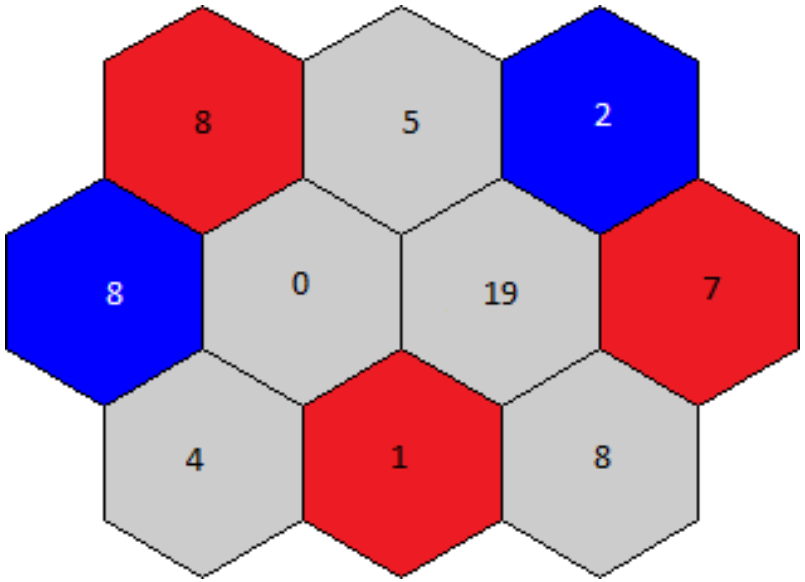
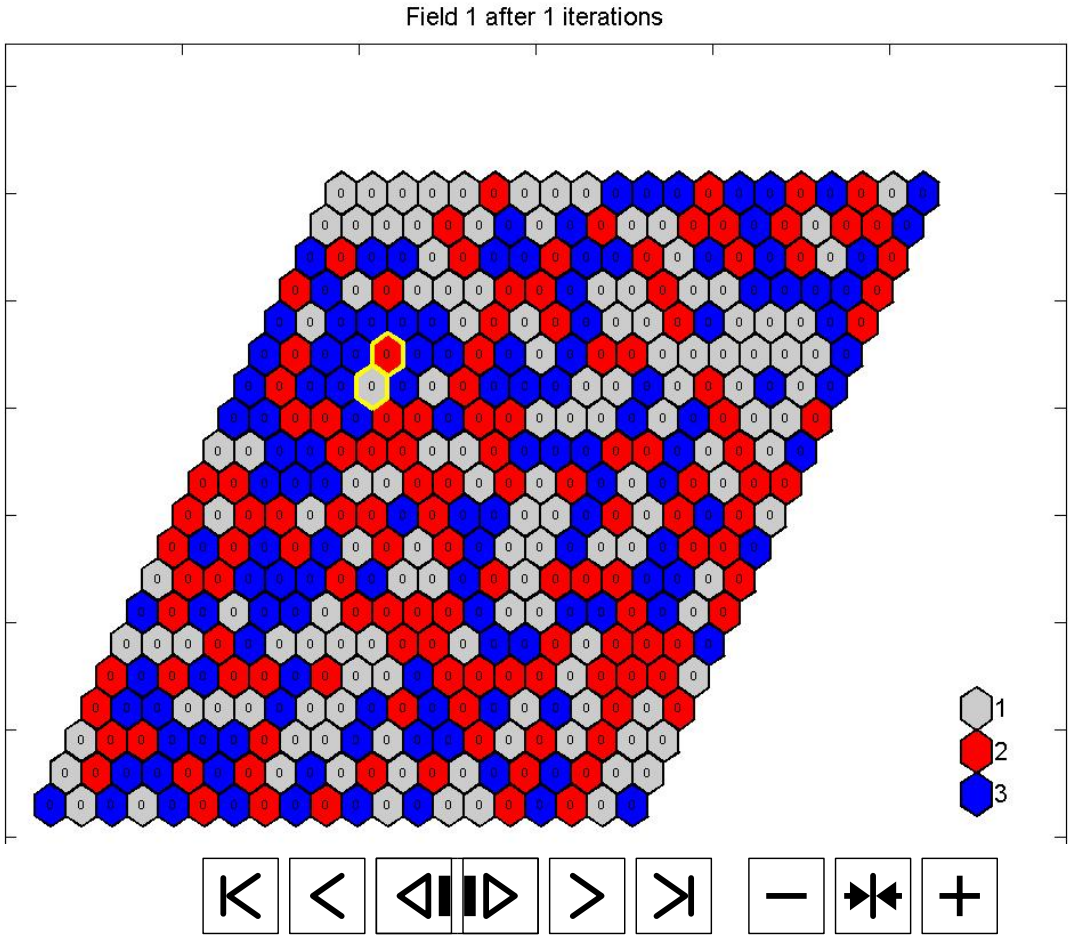


Numerical Interactions with Neighbour Support – Small Time Steps

$$\begin{array}{c} \\ \blacksquare \\ \blacksquare \\ \blacksquare \end{array} \begin{array}{c} \begin{array}{ccc} \blacksquare & \blacksquare & \blacksquare \end{array} \\ \begin{pmatrix} 0 & 1 & 0 \\ 3 & 0 & 0 \\ -2 & 2 & 0 \end{pmatrix} \end{array}$$



Numerical Interactions with Neighbour Support – Big Time Steps

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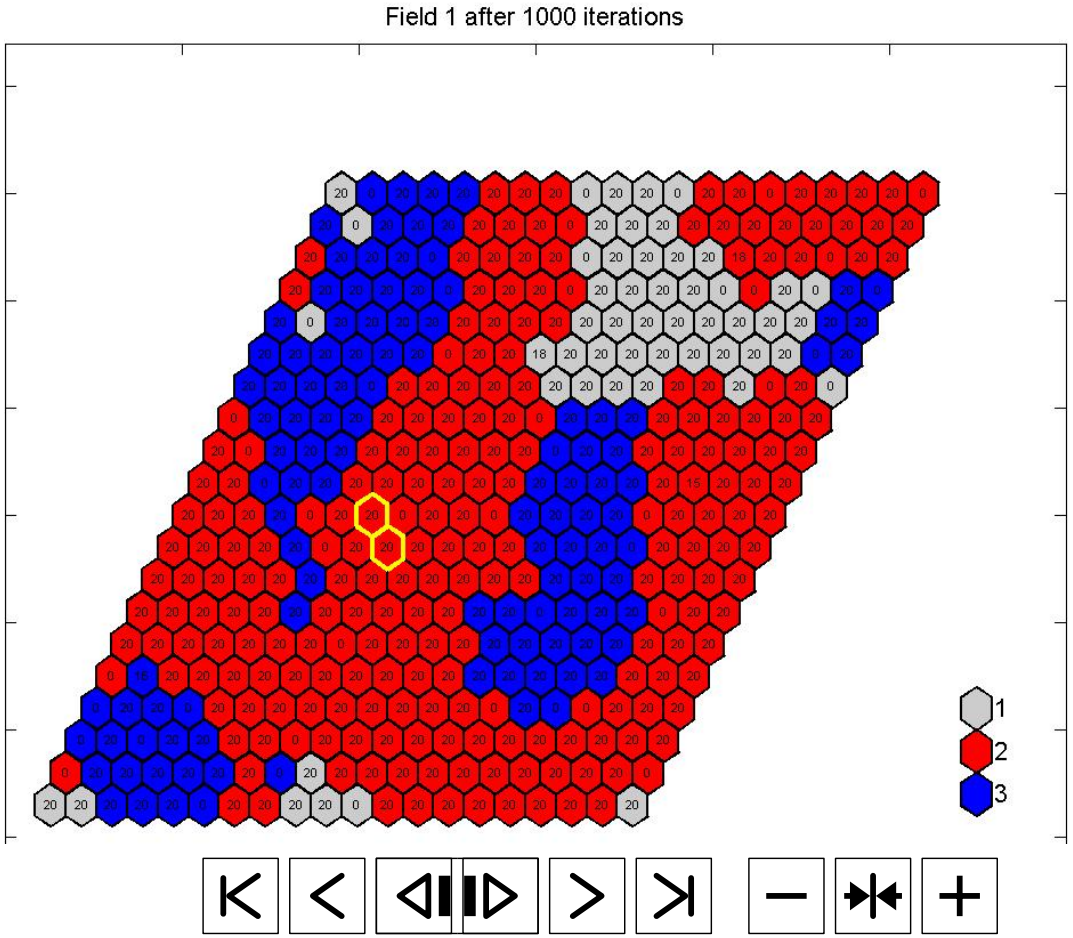
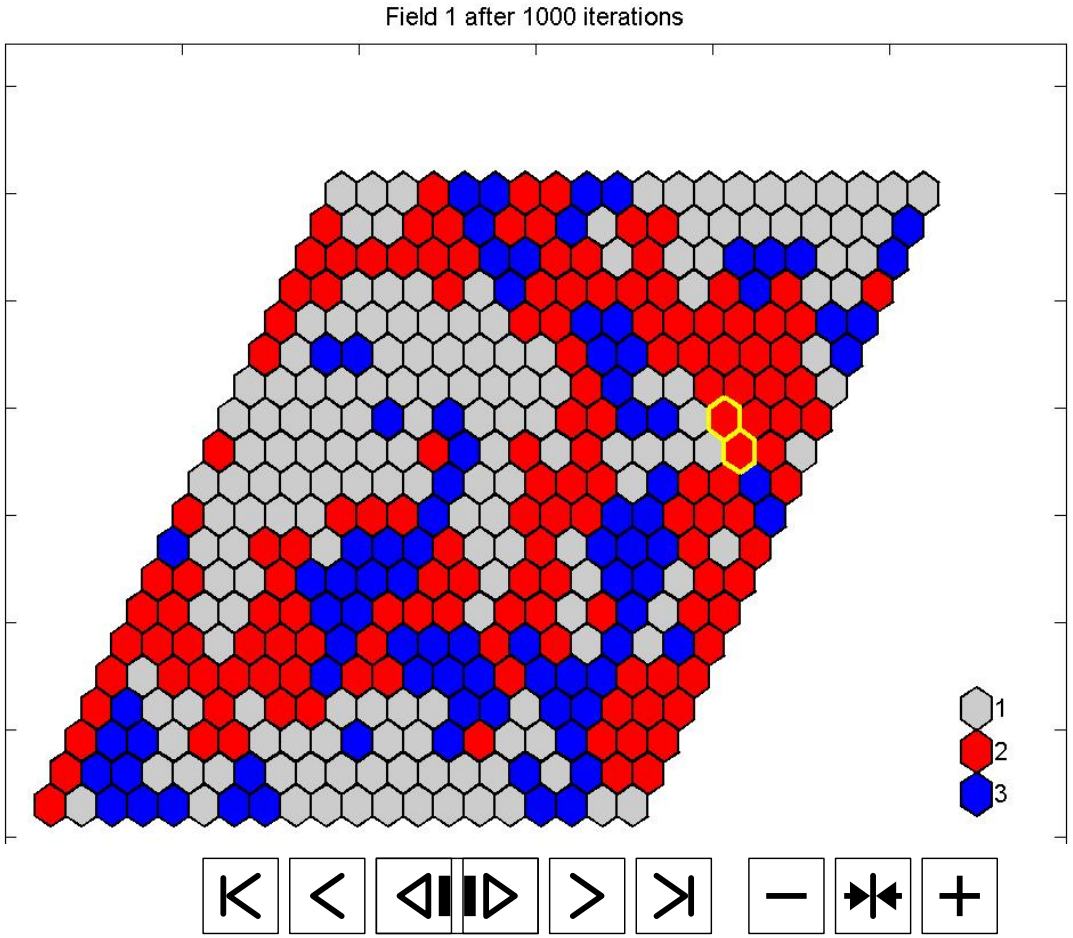


Recall: Local Interactions without Neighbour Support

For the *same* matrix

$$\begin{array}{c} \\ \blacksquare \\ \blacksquare \\ \blacksquare \end{array} \begin{array}{c} \begin{array}{ccc} \blacksquare & \blacksquare & \blacksquare \end{array} \\ \begin{pmatrix} 0 & 1 & 0 \\ 3 & 0 & 0 \\ -2 & 2 & 0 \end{pmatrix} \end{array}$$



Other Population Development Work

Other Population Development Work

- Studying ovipositioning behavior of parasitoid wasps

Other Population Development Work

- Studying ovipositioning behavior of parasitoid wasps
- Studying the interactions of phytoseiid mites

Other Population Development Work

- Studying ovipositioning behavior of parasitoid wasps
- Studying the interactions of phytoseiid mites
- Studying the development of obesity among people

Other Population Development Work

- Studying ovipositioning behavior of parasitoid wasps
- Studying the interactions of phytoseiid mites
- Studying the development of obesity among people
- One (or two) PhD projects starting September 2012

Thanks

Thank you for your attention!
Any comment is welcome!