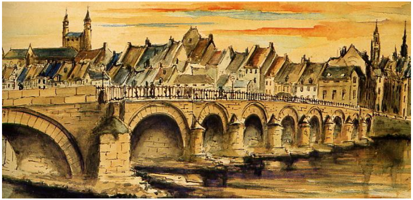
Network Characteristics and Efficient Coordination



Frank Thuijsman

joint work with

Abhimanyu Khan, Ronald Peeters, Philippe Uyttendaele

Maastricht University

Outline

Coordination Game

$$\begin{array}{c|cc|} & P & R \\ \hline P & a, a & b, c \\ R & c, b & d, d \\ \hline \end{array}$$

Assumptions:

1. $a > c, d > b$: pure equilibria (P, P) and (R, R);
2. $a > d$: payoff on P Pareto dominates payoff on R;
3. $c > b$: in case of miscoordination, R is safer.

Population of Players

Assumptions:

1. even number of players;
2. players are connected in (social) network;
3. at discrete stages $1, 2, 3, \ldots$ players are randomly matched to other players;
4. at each stage, each player chooses P or R by imitating neighbor with highest realized payoff;
5. neighbors of a player include the player himself.

The Imitation Rule

Please note:

1. A player imitates P, if *at least one* P neighbor was matched to a P node.

	P	R
P	$6,6$	$0,3$
R	$3,0$	$4,4$

The Imitation Rule

Please note:

1. A player imitates P, if *at least one* P neighbor was matched to a P node.
2. A player imitates R, if *all* P neighbors were matched to R nodes.

	P	R
P	$6,6$	$0,3$
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Goal of Study

We want to investigate the influence of network characteristics:

1. on convergence to the efficient outcome P;
2. on the speed of convergence to the efficient outcome P.

Related Work

- **Robson A and Vega-Redondo F** (1996). Efficient equilibrium selection in evolutionary games with random matching. *Journal of Economic Theory* 70 (1): 65-92.
- **Chen H-C, Chow Y and Wu L-C** (2013). Imitation, local interaction, and coordination. *International Journal of Game Theory* 42 (4): 1041-1057.
- **Alós-Ferrer C and Weidenholzer S** (2008). Contagion and efficiency. *Journal of Economic Theory* 143 (1): 251-274.
- **Khan A** (2013). Coordination under global random interaction and local imitation. Forthcoming *International Journal of Game Theory*.

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Two Types of Networks

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- **Scale-Free Networks**

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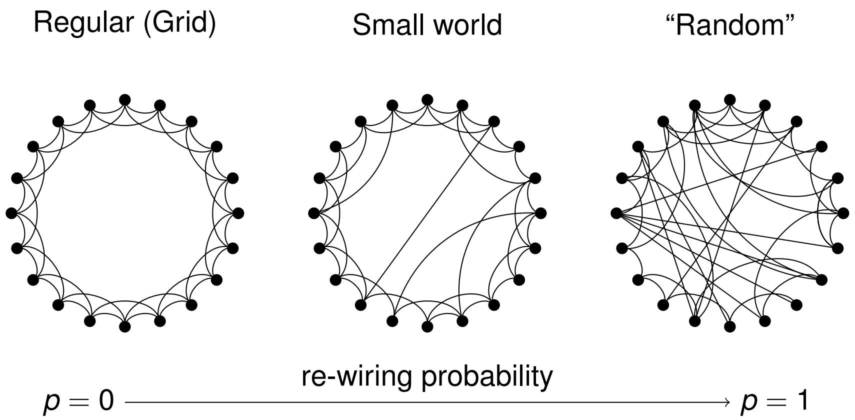
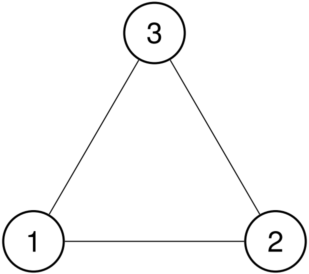
Small-World Networks



Scale-Free Networks

Method of construction



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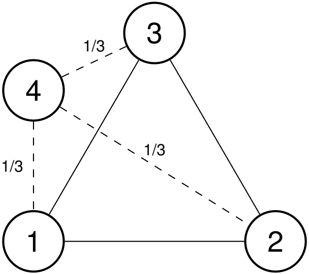

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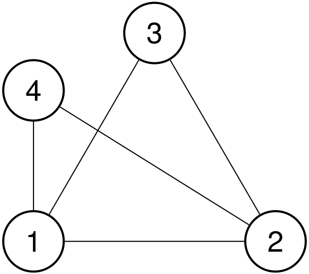

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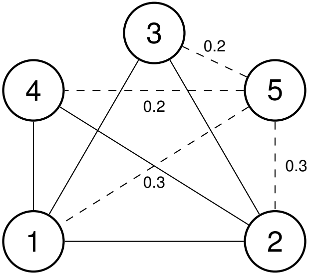

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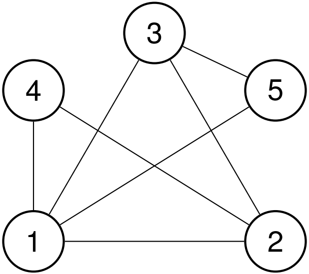

Networks of Different Size and Degree

	Small-World Networks	Scale-Free Networks
Parameter	Values	Values
Population size	100, 200 and 300	100, 200 and 300
Number of links per node	4, 6 and 8	4, 6 and 8 (approx.)
Relinking probability	0.00, 0.10, 0.20, 0.30, 0.40, 0.50, 0.60, 0.70, 0.80, 0.90 and 1.00	
Number of reconfigurations	100	100
Share of nodes seeded with strategy P	0.02, 0.05, 0.10, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40 and 0.45	0.02, 0.05, 0.10, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40 and 0.45
Number of reconfigurations (per network)	100	100
Number of initial configurations	9,900,000	900,000
Number of runs per initial configuration	100	100
Total number of runs	990,000,000	90,000,000

Network Simulations

For each of 10,800,000 initializations we ran 100 simulations.

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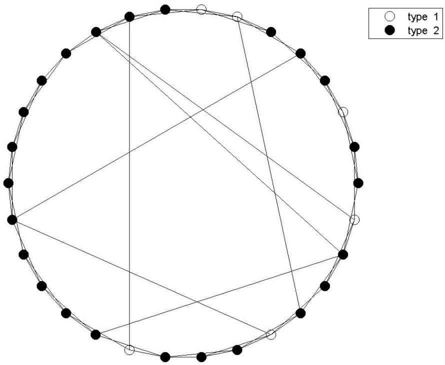
At each stage all nodes were randomly paired to other nodes.

At each stage, each node played the strategy that, among all its neighbors, did best at the previous stage.
Please note: each node is a neighbor of itself.

An Example on a Small-World Network



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P	$6, 6$	$0, 3$
R	$3, 0$	$4, 4$

Initially 20% P, type 1, white

Re-wiring prob. 0.2

Average Degree 4

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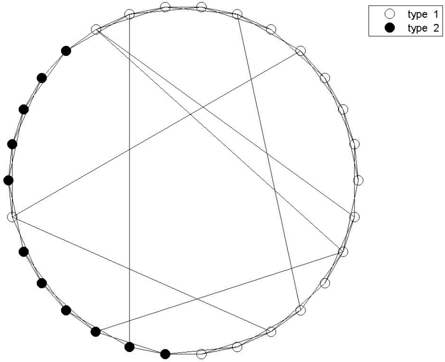

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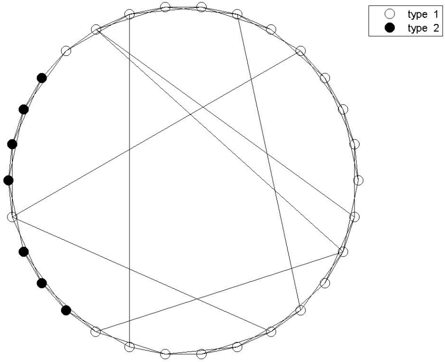

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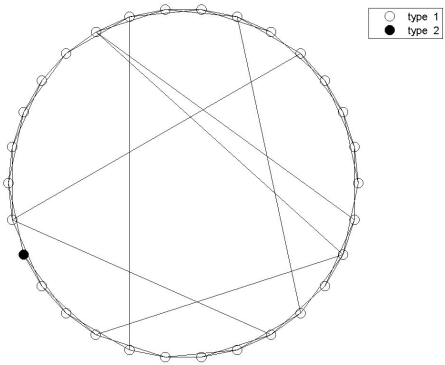

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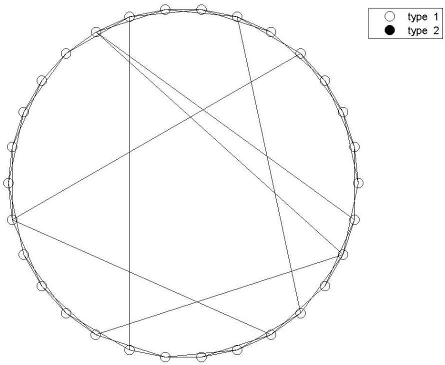

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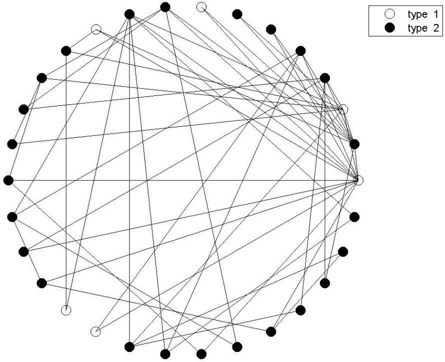

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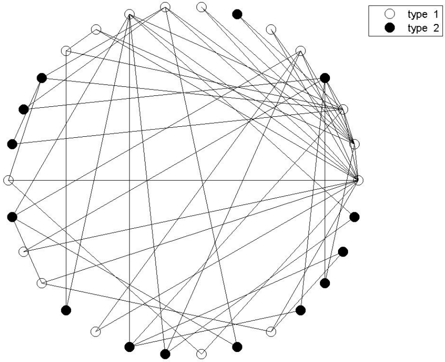

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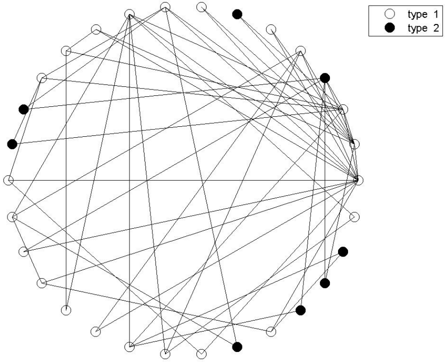

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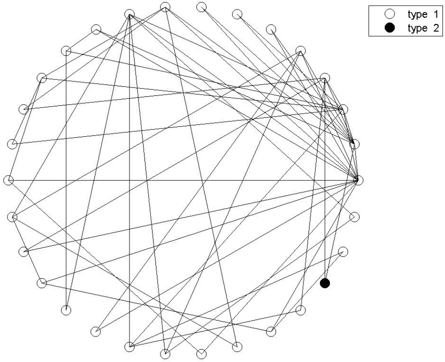

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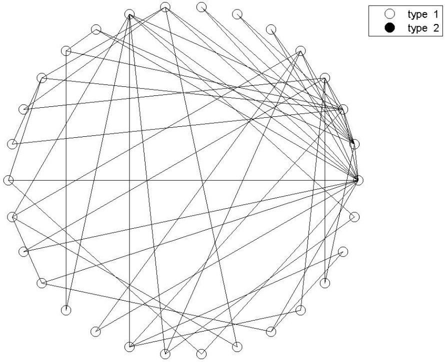

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Network Specific Characteristics (NSC)

- **Size**: *number of nodes*
- **Density**: *fraction of links used in network*
- **Degree**: *mean and s.d. of degree per node*
- **Power**: *mean and s.d. of power per node*

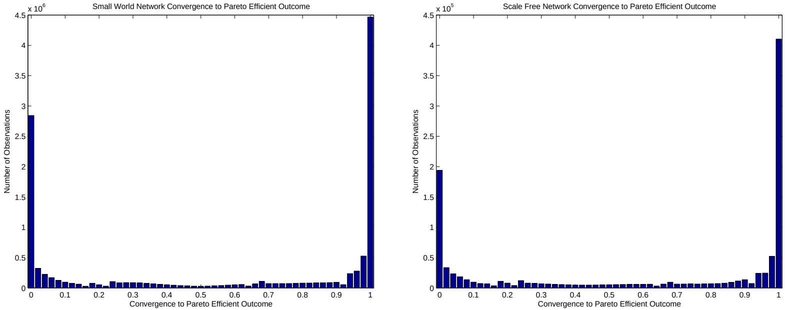
Initial Assignment of Strategies (IAS)

- **Share of** P **nodes**: *fraction of* P *nodes*
- **Degree of** P **nodes**: *mean and s.d. of degree per* P *node*
- **Power of** P **nodes**: *sum, mean and s.d.*
- **Segregation of** P **nodes**: *measure using random walks*
- **Segregation of** R **nodes**: *same*

Variables to Explain

- **Payoff Dominant Wins**: *proportion of P wins over 100 runs*
- **Mean Convergence Time to P wins**: *just what it says, but conditioned on the P equilibrium being reached at least 75% of 100 runs*

Number of Initializations for *P Wins* Proportions



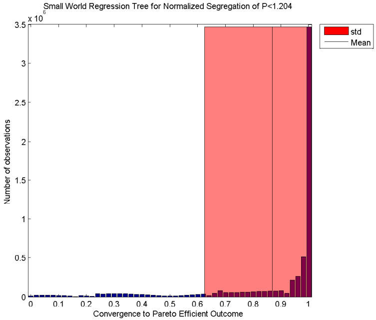
Regression Analysis on *P Wins*

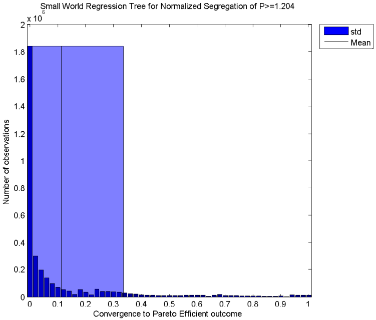
Using Size 200 and DegreeMean 6 as baseline:

	Small-world network		Scale-free network	
Variable	Coef.	Robust Std. Err.	Coef.	Robust Std. Err.
Size 100	−.018594	(.000166)	−.052502	(.000853)
Size 300	.006113	(.000145)	.019339	(.000503)
DegreeMean 4	−.108388	(.000160)	−.044391	(.000407)
DegreeMean 8	.056573	(.000143)	.017596	(.000436)
Share of *P* nodes	2.714903	(.000565)	2.397553	(.001748)
Degree of nodes: stdev	.012908	(.000148)		
Degree of *P* nodes: stdev			.014391	(.000112)
Power of *P* nodes: mean			3.910410	(.143479)
Power of *P* nodes: stdev	1.468833	(.072851)		
Segregation (norm.) of *P* nodes	−.029758	(.000102)	−.109114	(.000632)
Segregation (norm.) of *R* nodes	−.662857	(.003816)	−.151357	(.004433)
Constant	.698200	(.003861)	.305024	(.004809)
Number of obs.	9,900,000		900,000	
R-squared	0.8508		0.8562	

Small-World Classification Tree Analysis on *P Wins*

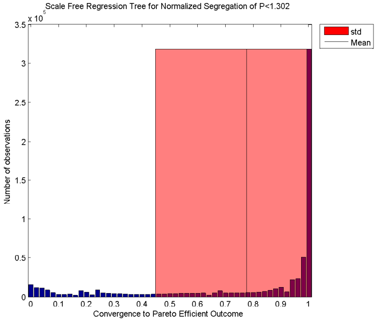
Selection	Convergence to P mean	std	Number of Initializations
Original dataset	59.9%	42.8%	(9,900,000)
Segregation (norm.) of P nodes < 1.204	86.7%	24.3%	(6,348,644)
Segregation (norm.) of P nodes ≥ 1.204	12.2%	22.3%	(3,551,356)

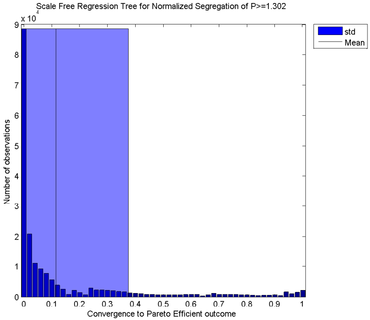




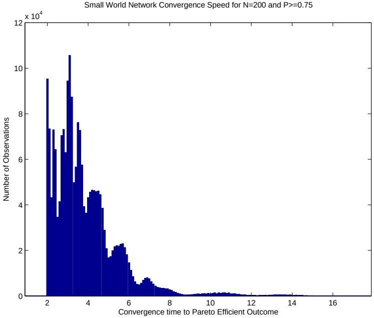
Scale-Free Classification Tree Analysis on *P Wins*

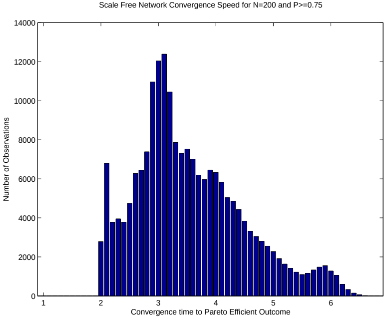
Selection	Convergence to *P* mean	std	Number of Initializations
Original dataset	63.7%	40.7%	(900,000)
Segregation (norm.) of *P* nodes < 1.302	77.6%	32.8%	(698,078)
Segregation (norm.) of *P* nodes ≥ 1.302	15.5%	25.9%	(201,992)





Number of Initializations for *Convergence Time to P Wins*





Regression Analysis on *Convergence Time to P Wins*

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	Small-world network		Scale-free network	
Variable	Coef.	Robust Std. Err.	Coef.	Robust Std. Err.
Size 100	.091336	(.001514)	−1.276585	(.006075)
Size 300	.021843	(.001079)	1.136900	(.002419)
DegreeMean 4	1.344045	(.001665)	.988007	(.001444)
DegreeMean 8	−.594475	(.000933)	−.483977	(.001010)
Share of *P* nodes	−11.159790	(.004769)	−.032329	(.000027)
Degree of nodes: stdev	−1.070549	(.001585)		
Degree of *P* nodes: stdev			−.053317	(.000462)
Power of *P* nodes: mean			−18.726130	(1.124271)
Power of *P* nodes: stdev	−247.282000	(.900051)		
Segregation (norm.) of *P* nodes	5.531019	(.014139)	.496119	(.010822)
Segregation (norm.) of *R* nodes	3.704960	(.022408)	1.447586	(.019315)
Constant	−.228171	(.034900)	4.141509	(.028758)
Number of obs.	5,265,575		501,081	
R-squared	0.7788		0.9014	

Comparing Scale-Free and Small-World Networks

1. In both cases **Size**, **DegreeMean**, **Share of *P* nodes** and **SD of *P* degree** have a positive effect on efficient coordination.
2. In both cases **Segregation of *P* nodes** and **Segregation of *R* nodes** have a negative effect on efficient coordination.
3. In both cases **Segregation of *P* nodes** is the most important variable to decide on convergence to *P* or to *R*.
4. In both cases **DegreeMean**, **Share of *P* nodes**, **Segregation of *P* nodes** and **Segregation of *R* nodes** have a positive effect on convergence time for those initializations with at least 0.75 convergence, while for both types of networks **Share of *P* nodes** has a negative effect.

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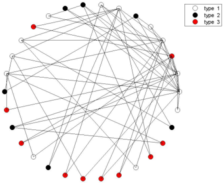
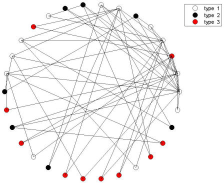
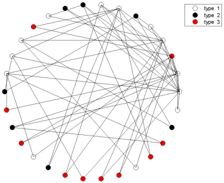
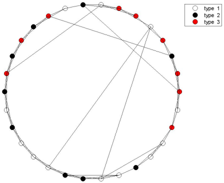
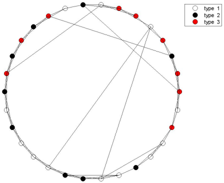
Experiments with More Types

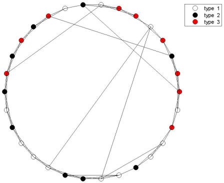












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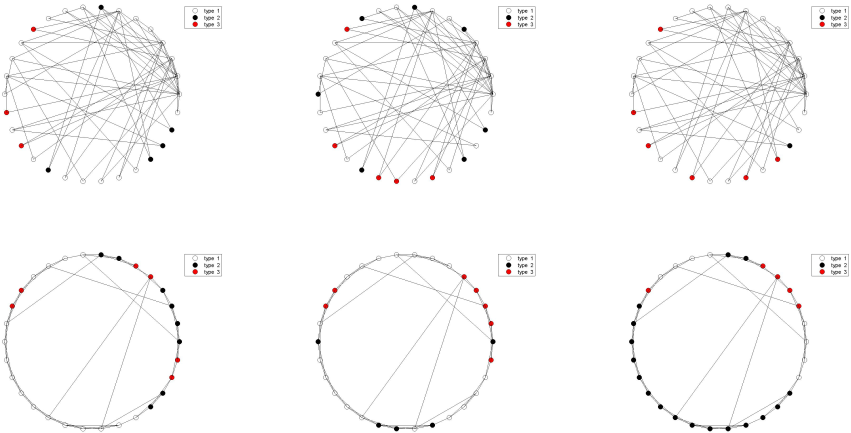

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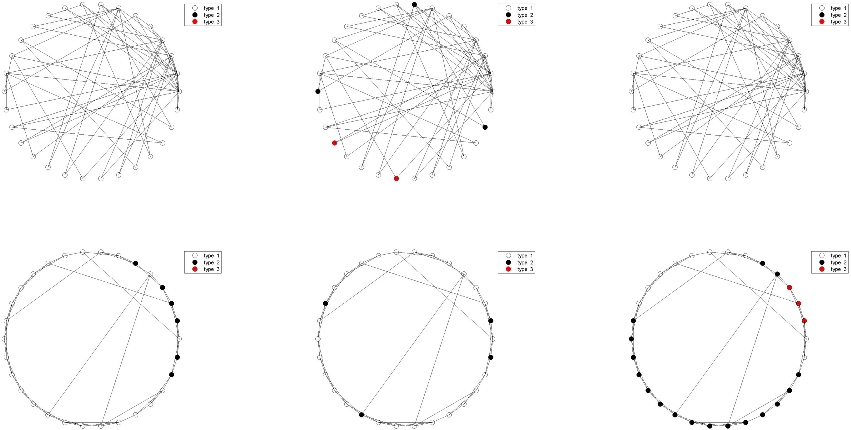

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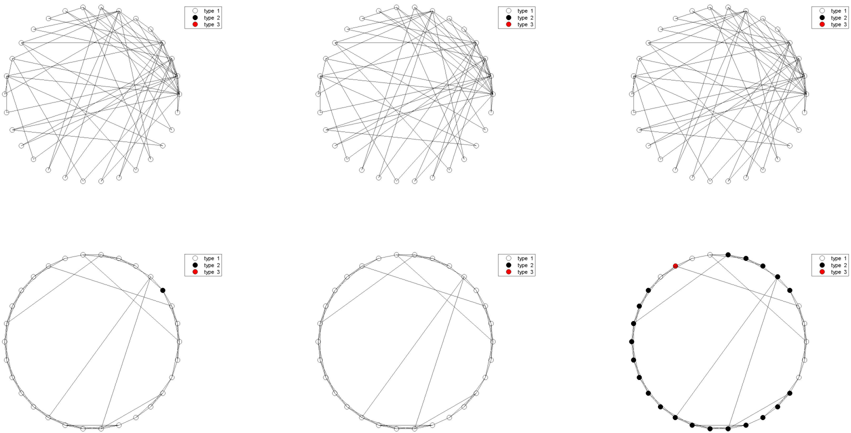

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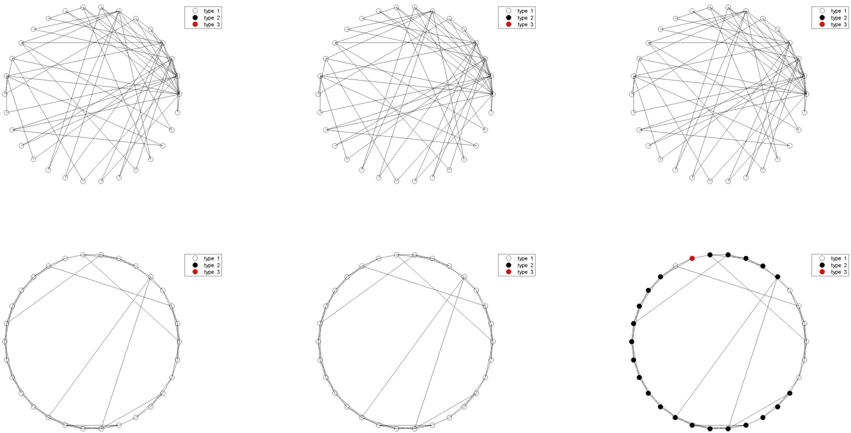
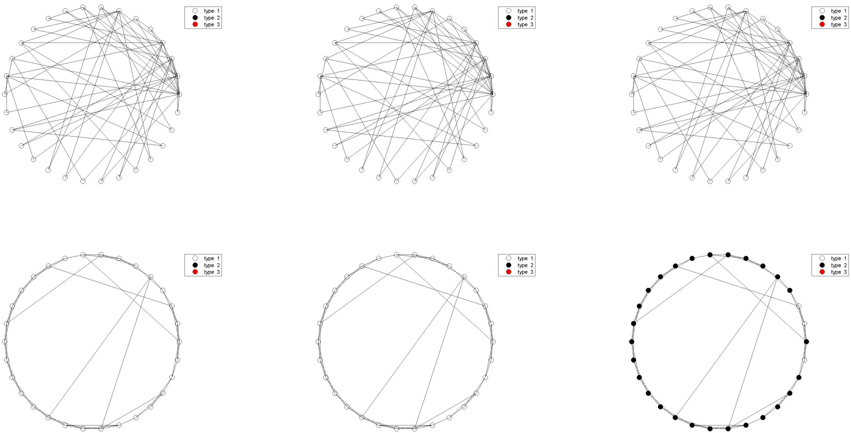
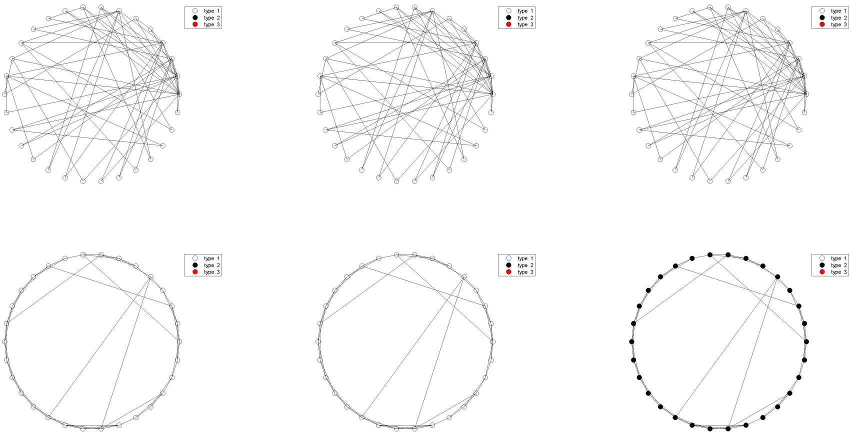


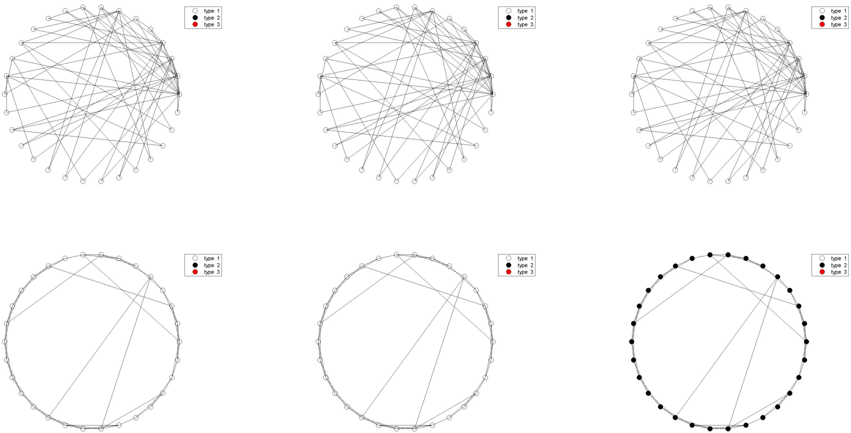
Experiments with More Types



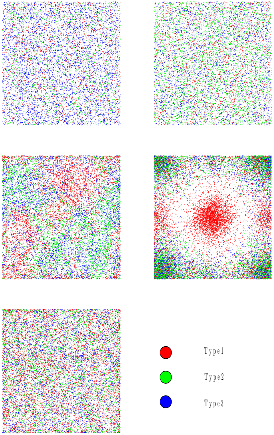
Experiments with More Types



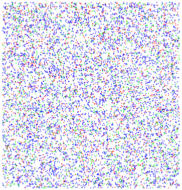
Experiments with More Types



New Experiments in Maastricht



Local **evolutionary** dynamics, where we experiment with the effect of interaction radius and offspring radius on segregation in a grid free world.



Thanks

Thank you for your attention!
Comments will be appreciated!

Paper is available and
Presentation will be posted on
https://dke.maastrichtuniversity.nl/f.thuijsman/