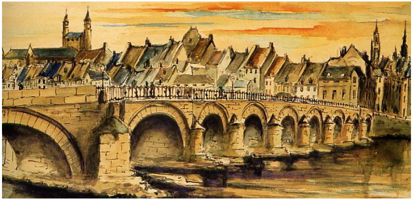
Network Characteristics and Efficient Coordination



Frank Thuijsman

joint work with

Abhimanyu Khan, Ronald Peeters, Philippe Uyttendaele

Outline

Coordination Game

	P	R
P	a, a	b, c
R	c, b	d, d

Assumptions:

1. $a > c, d > b$: pure equilibria (P, P) and (R, R);
2. $a > d$: payoff on P Pareto dominates payoff on R;
3. $c > b$: in case of mis-coordination, R is safer.

Population of Players

Assumptions:

1. even number of players;
2. players are connected in (social) network;
3. at discrete stages $1, 2, 3, \dots$ players are randomly matched to other players;
4. at each stage each player chooses P or R by imitating neighbor with highest payoff;
5. neighbors include self.

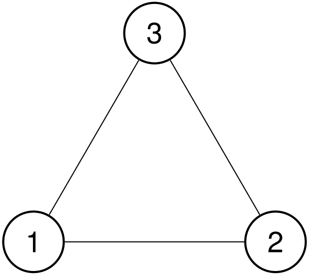
Goal of Study

We want to investigate the influence of network characteristics:

1. on convergence to the efficient outcome P;
2. on the speed of convergence to a homogeneous population.

Scale-Free Networks

Method of construction



Motivation:
Scale-free networks match empirical data on networks
Few nodes with high degree, many nodes with low degree.

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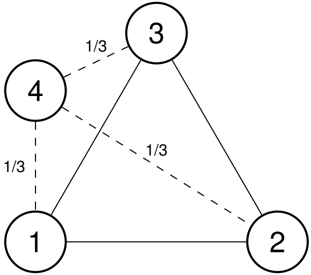

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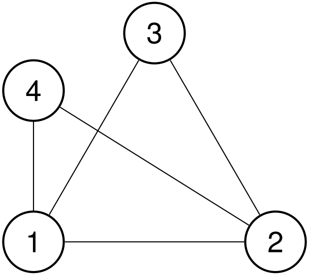

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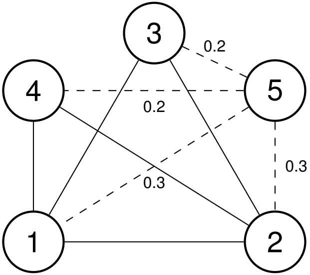

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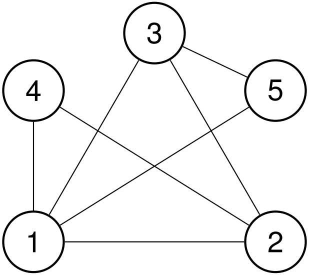

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Networks of Different Size and Degree

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For each of these we examined:

- 25 networks with average degree 4;
- 25 networks with average degree 6;
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This gave a total of 225 different networks.

Network Initializations

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Network Initializations

For each of 225 networks we used 300 random initializations.

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We used the following 12 fractions:
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This gave 67,500 different initializations.

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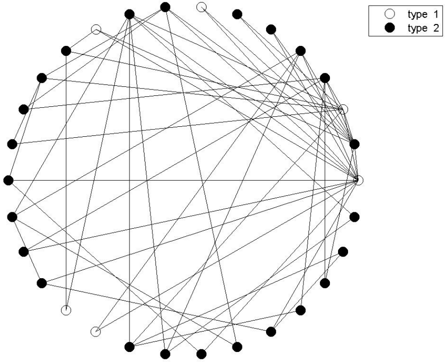
An Example on a Scale Free Network

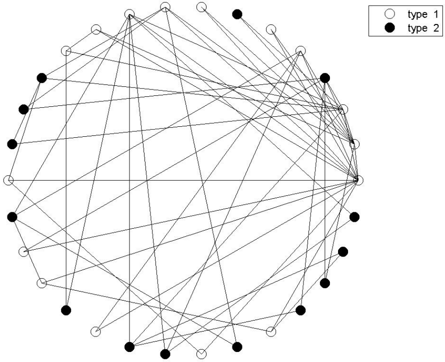


	P	R
P	6, 6	0, 3
R	3, 0	4, 4

Initially 20% P, type 1, white

Average Degree 4

An Example on a Scale Free Network



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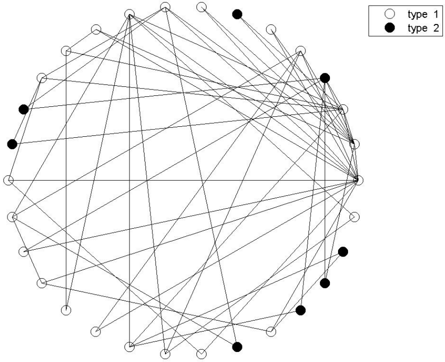

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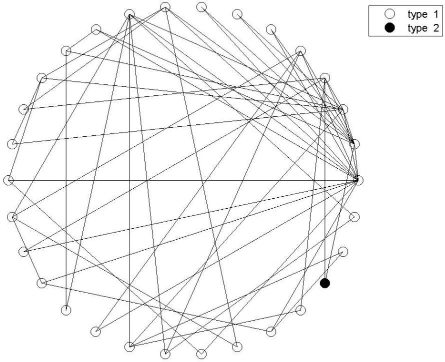

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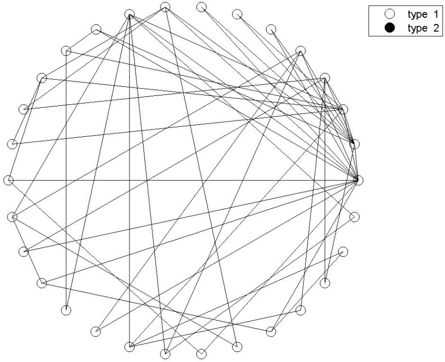

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Network Specific Characteristics (NSC)

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- *Density*:
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Initial Assignment of Strategies (IAS)

- *Share of P nodes*:
- *Degree of P nodes*:
- *Power of P nodes*:
- *Segregation of P nodes*:
- *Segregation of R nodes*:

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- *Degree of P nodes*: mean and s.d. of degree per *P* node
- *Power of P nodes*: sum, mean and s.d.
- *Segregation of P nodes*: measure using random walks
- *Segregation of R nodes*: same

Variables to Explain

- *Payoff Dominant Wins*:
- *Mean Convergence Time*:

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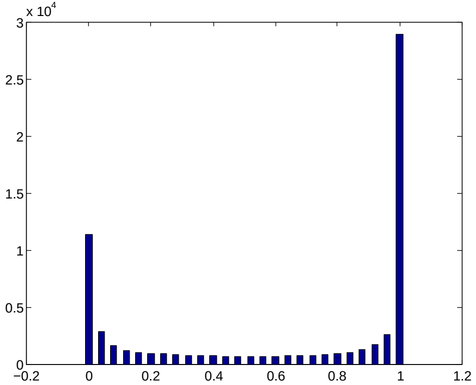
Each of these is measured over 25 runs for any specific choice of initialized network.

Number of Initializations for P Wins Proportions

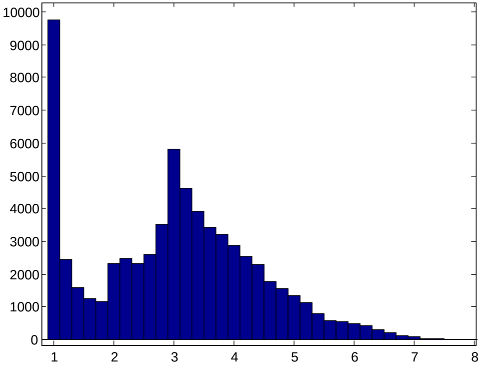


Regression Analysis on *Payoff Dominant Wins*

For the Scale Free Networks Examined:

Variable	Coef.	Effect
Size	.000166	positive
Degree: mean	.013205	positive
Share of *P* nodes	2.175143	positive
Degree of *P* nodes: stdev	.012700	positive
Segregation (norm.) of *P* nodes	-.053167	negative
Segregation (norm.) of *R* nodes	-.107330	negative
Constant	.121343	—
Number of obs.	67,500	
R-squared	0.8402	

Number of Initializations for *Convergence Time*



Regression Analysis on *Convergence Time*

For the Scale Free Networks Examined:

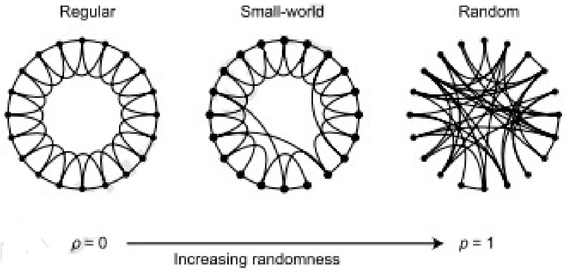
Variable	Coef.	Effect
Size	.002208	positive
Degree: mean	-.379215	negative
Share of P nodes	.302001	positive
Degree of P nodes: stdev	.113038	positive
Segregation (norm.) of P nodes	-.806758	negative
Segregation (norm.) of R nodes	1.765975	positive
Constant	3.551547	—
Number of obs.	67,500	
R-squared	0.4596	

Classification Tree Analysis on *Payoff Dominant Wins*

For the Scale Free Networks Examined:

Selection	Convergence to P mean	std	Number of observations
Original dataset	63.2%	42.2%	(65,625)
Segregation (norm.) of P nodes < 1.313	79.7%	32.2%	(49,742)
Segregation (norm.) of P nodes ≥ 1.313	11.2%	23.0%	(15,883)

Small World Networks



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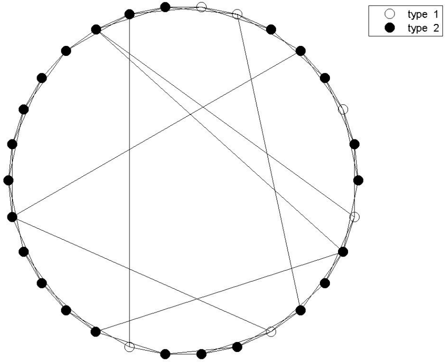

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Rewiring prob. 0,2

Average Degree 4

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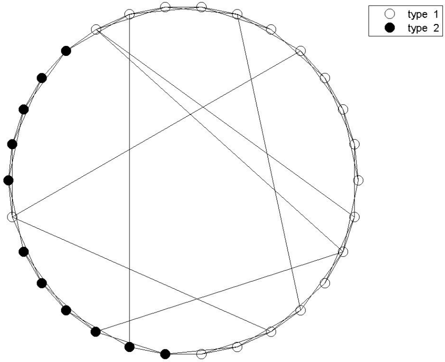

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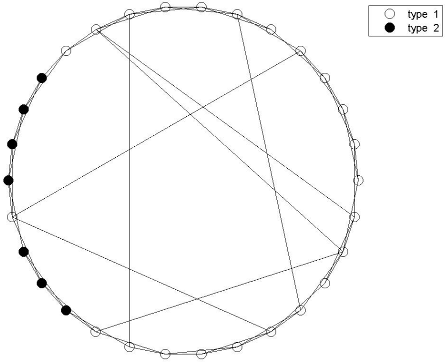

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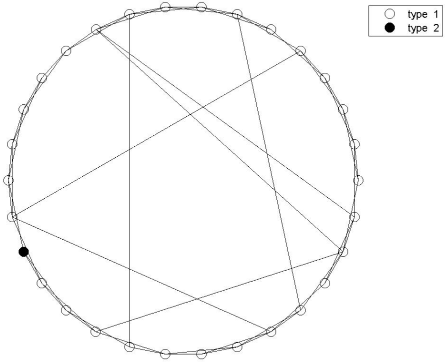

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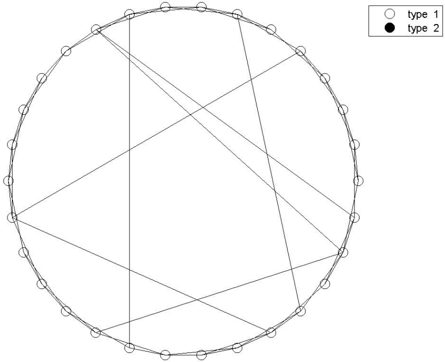

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This gave a total of 90 different networks.

Network Initializations

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And re-wiring probabilities: 0, 0.2, 0.4, 0.6, 0.8, 1

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This gave 64,800 different initializations.

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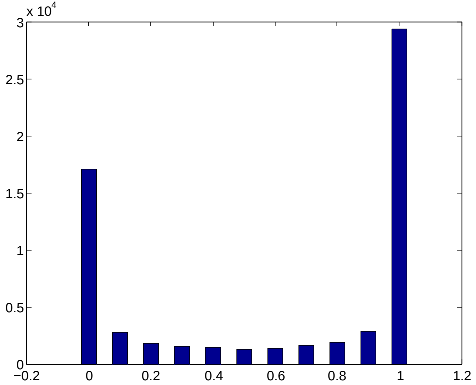
Number of Initializations for P Wins Proportions for SWN

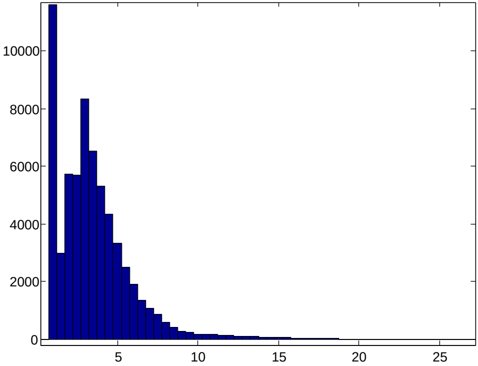


Small World Regression Analysis on *Payoff Dominant Wins*

For the Small Data Set of Small World Networks Examined:

Variable	Coef.	Effect
Size	.000085	positive
Degree: mean	.033065	positive
Degree: stdev	-.005834	negative
Share of *P* nodes	2.456533	positive
Degree of *P* nodes: stdev	.015220	positive
Segregation (norm.) of *P* nodes	-.014965	negative
Segregation (norm.) of *R* nodes	-.931433	negative
Constant	.757578	—
Number of obs.	64,800	
R-squared	0.8261	

Number of Initializations for *Convergence Time* for SWN



Small World Regression Analysis on *Convergence Time*

For the Small Data Set of Small World Networks Examined:

Variable	Coef.	Effect
Size	.004134	positive
Degree: mean	-.256099	negative
Degree: stdev	-1.742573	negative
Share of P nodes	-.590169	negative
Segregation (norm.) of P nodes	-.954044	negative
Segregation (norm.) of R nodes	-5.030866	negative
Constant	13.109070	—
Number of obs.	64,799	
R-squared	0.3382	

Small World Classification Tree Analysis

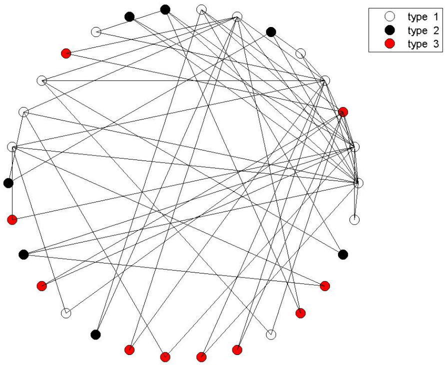
For the Small Data Set of Small World Networks Examined:

Selection	Convergence to P mean	std	Number of observations
Original dataset	59.8%	40.2%	(63,000)
Segregation (norm.) of P nodes < 1.210	88.3%	24.2%	(39,947)
Segregation (norm.) of P nodes ≥ 1.210	10.5%	22.6%	(23,053)

Comparison of Results for Scale-Free and Small World Networks

1. In both cases *Size* and *Share of P nodes* have a positive effect on efficient coordination.
2. In both cases *Segregation of P nodes* and *Segregation of R nodes* have a negative effect on efficient coordination.
3. In both cases *Segregation of P nodes* is the most important variable to decide on convergence to *P* or to *R*.

A Scale Free Network with 3 Types



	P	R	S
P	6, 6	0, 3	0, 1
R	3, 0	4, 4	1, 2
S	1, 0	2, 1	3, 3

Initial distr.
(0.4; 0.2; 0.2)

Average Degree 4

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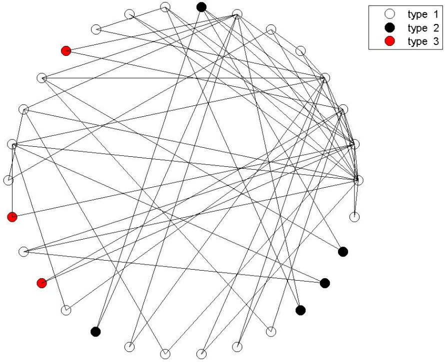

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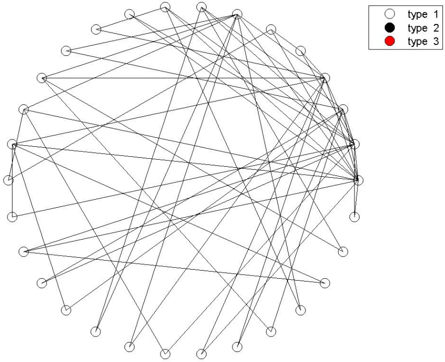

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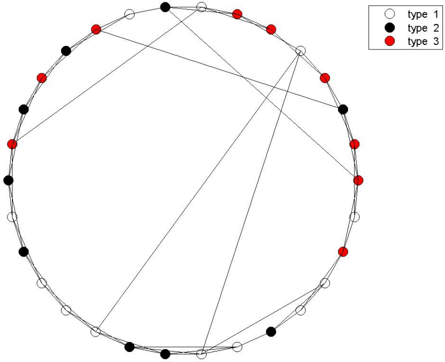

	P	R	S
P	$6,6$	$0,3$	$0,1$
R	$3,0$	$4,4$	$1,2$
S	$1,0$	$2,1$	$3,3$

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Rewiring prob. 0,2

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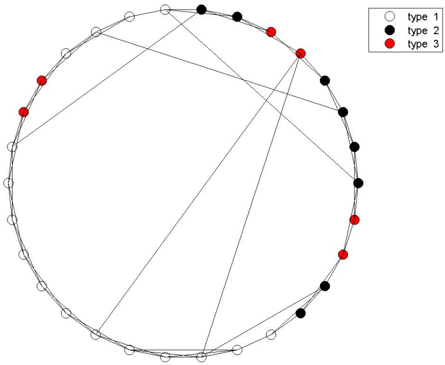

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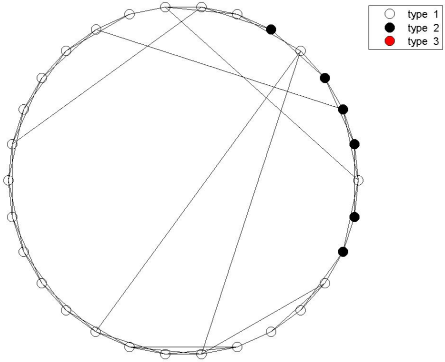

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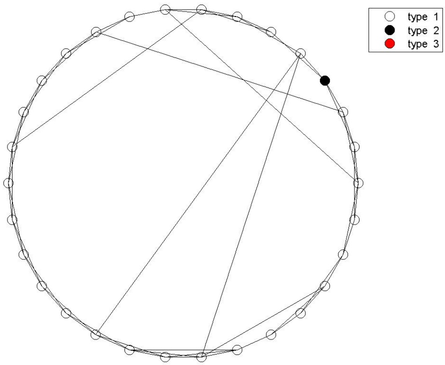

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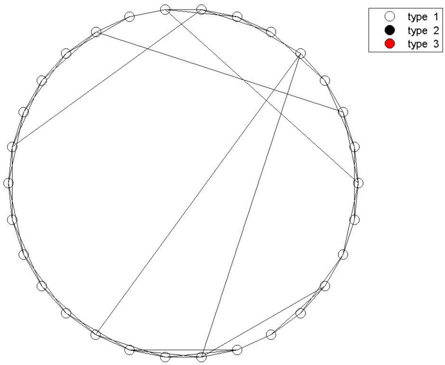
A Small World Network with 3 Types



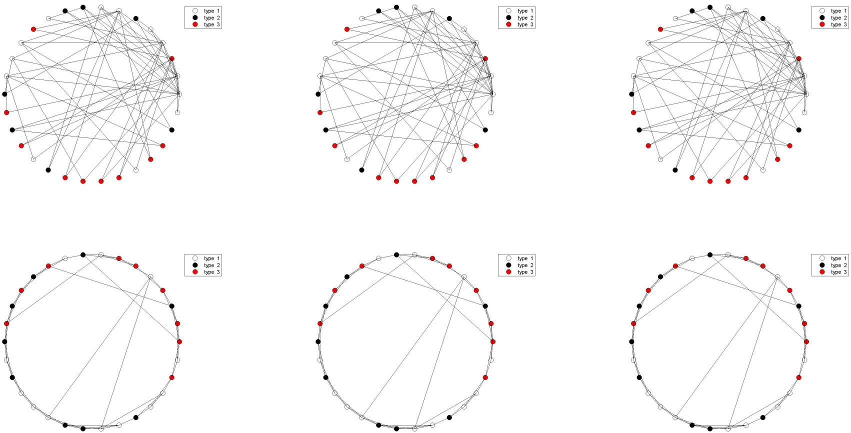
	P	R	S
P	6, 6	0, 3	0, 1
R	3, 0	4, 4	1, 2
S	1, 0	2, 1	3, 3

Initial distr.
(0.4; 0.2; 0.2)

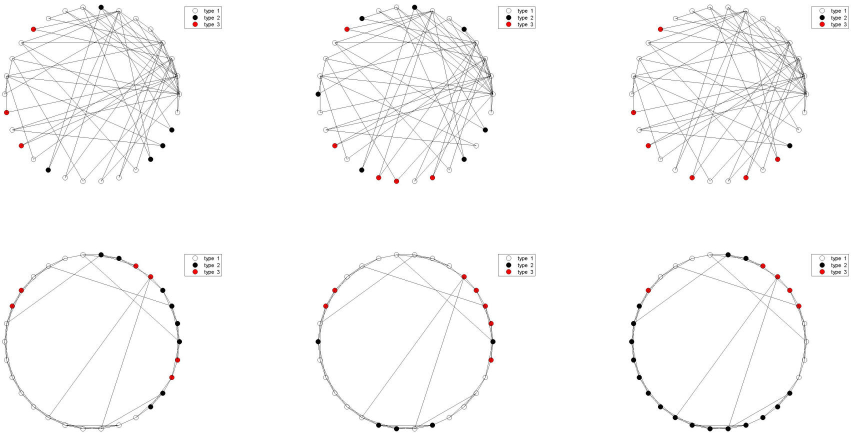
Rewiring prob. 0,2

Average Degree 4

Six Runs in Parallel



Six Runs in Parallel



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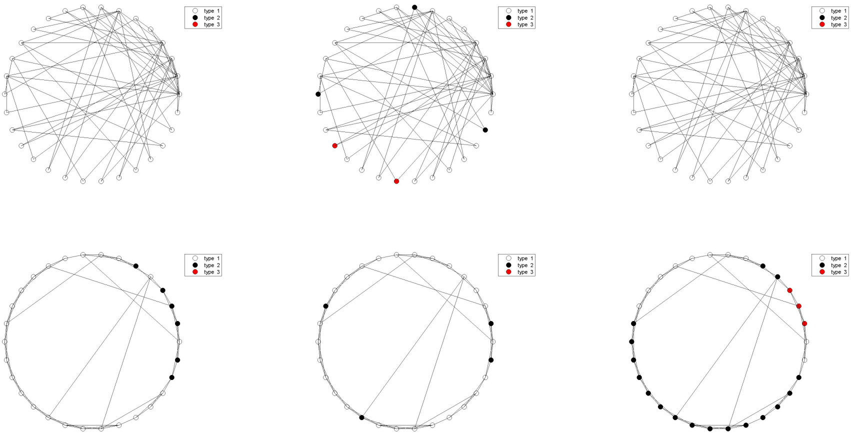
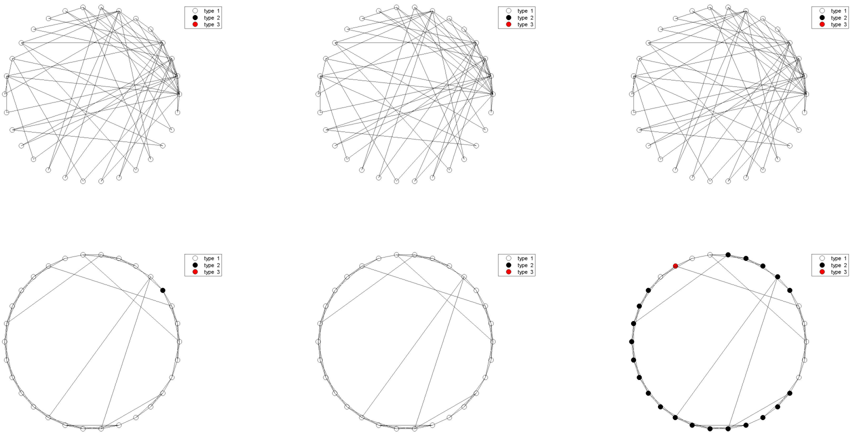


Six Runs in Parallel



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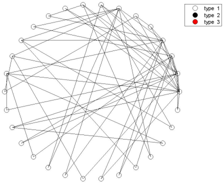
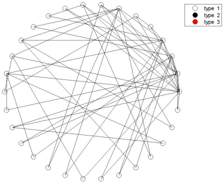
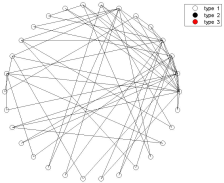
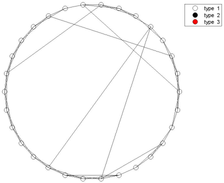
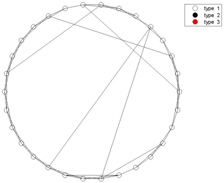
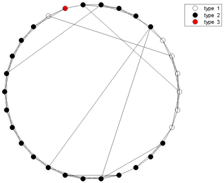




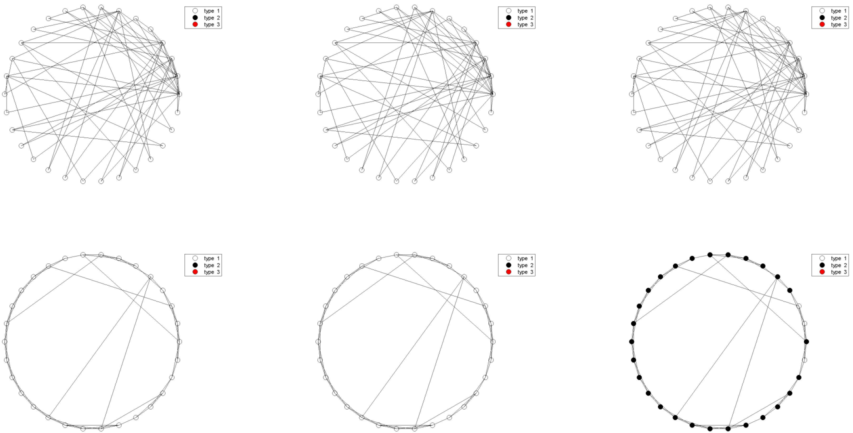








Six Runs in Parallel



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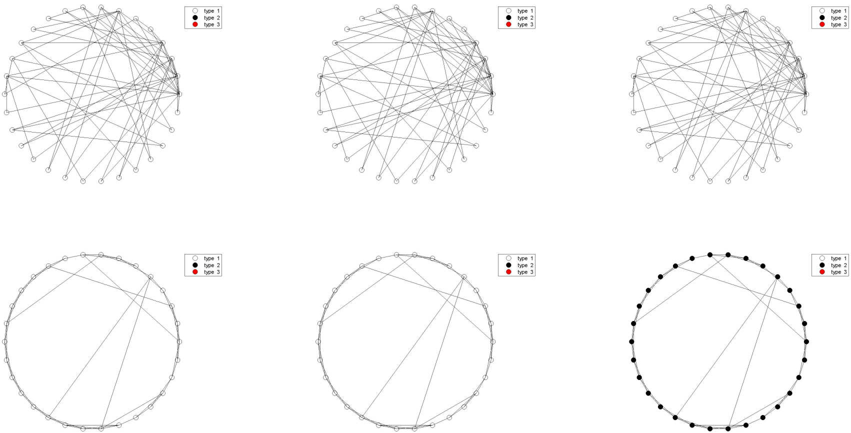
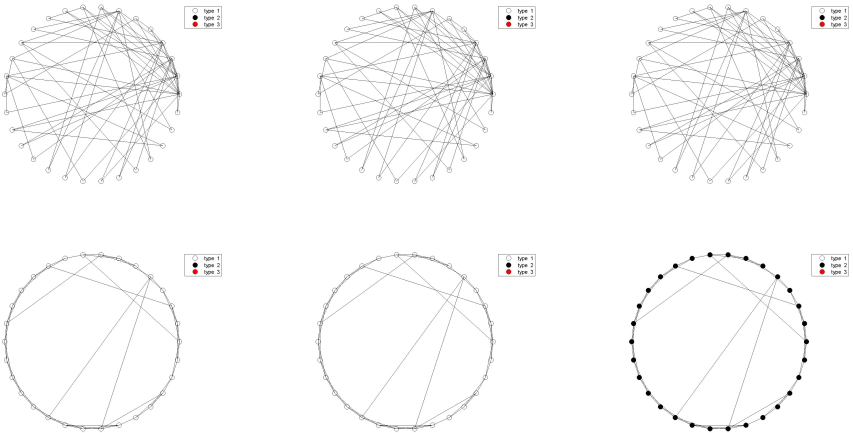


Six Runs in Parallel



Thanks

Thank you for your Attention!
Comments will be appreciated!

Presentation and paper will soon be available at
https://dke.maastrichtuniversity.nl/f.thuijsman/