Network Characteristics and Efficient Coordination

Frank Thuijsman

joint work with

Abhimanyu Khan, Ronald Peeters, Philippe Uyttendaele

Maastricht University
Introduction

A Simulation Study

Characteristics

Results

Other Networks

More Types
Coordination Game

Assumptions:
1. \( a > c, d > b \): pure equilibria \((P, P)\) and \((R, R)\);
2. \( a > d \): payoff on \(P\) Pareto dominates payoff on \(R\);
3. \( c > b \): in case of mis-coordination, \(R\) is safer.
Population of Players

Assumptions:

1. even number of players;
2. players are connected in (social) network;
3. at discrete stages 1, 2, 3, ... players are randomly matched to other players;
4. at each stage each player chooses $P$ or $R$ by imitating neighbor with highest realized payoff;
5. neighbors include self.
Goal of Study

We want to investigate the influence of network characteristics:

1. on convergence to the efficient outcome $P$;
2. on the speed of convergence to a homogeneous population.
Scale-Free Networks

Method of construction

Motivation:
Scale-free networks match empirical data on networks
Few nodes with high degree, many nodes with low degree.
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Few nodes with high degree, many nodes with low degree.
Scale-Free Networks

Method of construction

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Scale-free networks match empirical data on networks
Few nodes with high degree, many nodes with low degree.
We created scale-free networks with 100, 200 and 300 nodes.
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For each of these we examined:

- 100 networks with average degree 4;
- 100 networks with average degree 6;
- 100 networks with average degree 8.
We created scale-free networks with 100, 200 and 300 nodes.

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- 100 networks with average degree 4;
- 100 networks with average degree 6;
- 100 networks with average degree 8.

This gave a total of 900 different networks.
For each of 900 networks we used 1,200 random initializations.
Network Initializations

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These initializations were made by creating 100 random colorings for each of 12 fixed fractions of $P$ nodes.
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These initializations were made by creating 100 random colorings for each of 12 fixed fractions of $P$ nodes.

We used the following 12 fractions:
0.01, 0.02, 0.05, 0.10, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50.
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This gave 1,080,000 different initializations.
Network Simulations

For each of 1,080,000 initializations we ran 100 simulations.
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This gave a total of 108,000,000 simulations.

Each simulation ran until all nodes were of the same type.

At each stage all nodes were randomly paired to other nodes.

Each node played the strategy that did best among its neighbors (each node is one of its own neighbors).
An Example on a Scale Free Network

Initially 20% $P$, type 1, white

Average Degree 4
An Example on a Scale Free Network

Initially 20% $P$, type 1, white

Average Degree 4

<table>
<thead>
<tr>
<th></th>
<th>$P$</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$</td>
<td>6, 6</td>
<td>0, 3</td>
</tr>
<tr>
<td>$R$</td>
<td>3, 0</td>
<td>4, 4</td>
</tr>
</tbody>
</table>
An Example on a Scale Free Network

Initially 20% $P$, type 1, white

Average Degree 4

$$\begin{array}{ccc}
  P & R \\
  6, 6 & 0, 3 \\
  3, 0 & 4, 4 \\
\end{array}$$
An Example on a Scale Free Network

Initially 20% $P$, type 1, white

Average Degree 4
An Example on a Scale Free Network

Initially 20% $P$, type 1, white

Average Degree 4

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Network Specific Characteristics (NSC)

- **Size:**
- **Density:**
- **Degree:**
- **Power:**
### Network Specific Characteristics (NSC)

- **Size:** number of nodes
- **Density:**
- **Degree:**
- **Power:**
Network Specific Characteristics (NSC)

- **Size**: number of nodes
- **Density**: fraction of links used in network
- **Degree**:
- **Power**:
**Network Specific Characteristics (NSC)**

- **Size**: number of nodes
- **Density**: fraction of links used in network
- **Degree**: mean and s.d. of degree per node
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Network Specific Characteristics (NSC)

- **Size**: number of nodes
- **Density**: fraction of links used in network
- **Degree**: mean and s.d. of degree per node
- **Power**: mean and s.d. of power per node
Initial Assignment of Strategies (IAS)

- Share of P nodes:
- Degree of P nodes:
- Power of P nodes:
- Segregation of P nodes:
- Segregation of R nodes:
Initial Assignment of Strategies (IAS)

- **Share of P nodes**: fraction of P nodes
- **Degree of P nodes**: 
- **Power of P nodes**: 
- **Segregation of P nodes**: 
- **Segregation of R nodes**: 

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Initial Assignment of Strategies (IAS)

- **Share of P nodes**: fraction of $P$ nodes
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- **Segregation of R nodes**:
Initial Assignment of Strategies (IAS)

- **Share of P nodes**: fraction of $P$ nodes
- **Degree of P nodes**: mean and s.d. of degree per $P$ node
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- **Segregation of P nodes**:
- **Segregation of R nodes**:
Initial Assignment of Strategies (IAS)

- **Share of P nodes**: fraction of P nodes
- **Degree of P nodes**: mean and s.d. of degree per P node
- **Power of P nodes**: sum, mean and s.d.
- **Segregation of P nodes**: measure using random walks
- **Segregation of R nodes**: same
Variables to Explain

- Payoff Dominant Wins:
- Mean Convergence Time:
Variables to Explain

- **Payoff Dominant Wins**: proportion of $P$ wins
- **Mean Convergence Time**: 
Variables to Explain

- **Payoff Dominant Wins**: proportion of $P$ wins
- **Mean Convergence Time**: just what it says
Variables to Explain

- **Payoff Dominant Wins:** proportion of $P$ wins
- **Mean Convergence Time:** just what it says

Each of these is measured over 100 runs for any specific choice of initialized network.
Number of Initializations for $P$ Wins Proportions

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Regression Analysis on *Payoff Dominant Wins*

For the Scale Free Networks Examined:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coef.</th>
<th>Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size</td>
<td>0.000117</td>
<td>positive</td>
</tr>
<tr>
<td>Degree: mean</td>
<td>0.011441</td>
<td>positive</td>
</tr>
<tr>
<td>Share of $P$ nodes</td>
<td>2.182151</td>
<td>positive</td>
</tr>
<tr>
<td>Degree of $P$ nodes: stddev</td>
<td>0.014224</td>
<td>positive</td>
</tr>
<tr>
<td>Power of $P$ nodes: stddev</td>
<td>-2.428675</td>
<td>negative</td>
</tr>
<tr>
<td>Segregation (norm.) of $P$ nodes</td>
<td>-0.053563</td>
<td>negative</td>
</tr>
<tr>
<td>Segregation (norm.) of $R$ nodes</td>
<td>-0.134324</td>
<td>negative</td>
</tr>
<tr>
<td>Constant</td>
<td>0.171971</td>
<td>—</td>
</tr>
<tr>
<td>Number of obs.</td>
<td>1,080,000</td>
<td></td>
</tr>
<tr>
<td>R-squared</td>
<td>0.8478</td>
<td></td>
</tr>
</tbody>
</table>
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### Regression Analysis on *Convergence Time*

For the Scale Free Networks Examined:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coef.</th>
<th>Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size</td>
<td>0.00233</td>
<td>positive</td>
</tr>
<tr>
<td>Degree: mean</td>
<td>-0.37935</td>
<td>negative</td>
</tr>
<tr>
<td>Share of $P$ nodes</td>
<td>0.30642</td>
<td>positive</td>
</tr>
<tr>
<td>Degree of $P$ nodes: stdev</td>
<td>0.10754</td>
<td>positive</td>
</tr>
<tr>
<td>Power of $P$ nodes: stdev</td>
<td>4.10292</td>
<td>positive</td>
</tr>
<tr>
<td>Segregation (norm.) of $P$ nodes</td>
<td>-0.81094</td>
<td>negative</td>
</tr>
<tr>
<td>Segregation (norm.) of $R$ nodes</td>
<td>1.66375</td>
<td>positive</td>
</tr>
<tr>
<td>Constant</td>
<td>3.64648</td>
<td>—</td>
</tr>
</tbody>
</table>

Number of obs. 1,080,000  
R-squared 0.4691
Classification Tree Analysis on *Payoff Dominant Wins*

For the Scale Free Networks Examined:

<table>
<thead>
<tr>
<th>Selection</th>
<th>Convergence to $P$</th>
<th>Number of Initializations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original dataset</td>
<td>63.2%</td>
<td>42.0%</td>
</tr>
<tr>
<td>Segregation (norm.) of $P$ nodes $&lt; 1.302$</td>
<td>80.1%</td>
<td>31.7%</td>
</tr>
<tr>
<td>Segregation (norm.) of $P$ nodes $\geq 1.302$</td>
<td>12.0%</td>
<td>23.7%</td>
</tr>
</tbody>
</table>
Small World Networks

- Regular (Grid)
- Small world
- “Random”

$p = 0$ (re-wiring probability) \rightarrow p = 1$

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An Example on a Small World Network

Initially 20% \( P \), type 1, white

Re-wiring prob. 0.2

Average Degree 4
An Example on a Small World Network

Initially 20% $P$, type 1, white

Re-wiring prob. 0.2

Average Degree 4
An Example on a Small World Network

Initially 20% $P$, type 1, white

Re-wiring prob. 0.2

Average Degree 4

- $P$:
  - 6, 6
  - 0, 3

- $R$:
  - 3, 0
  - 4, 4

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An Example on a Small World Network

Initially 20% $P$, type 1, white

Re-wiring prob. 0.2

Average Degree 4
We created small world networks with 100, 200 and 300 nodes.
Networks of Different Size and Degree

We created small world networks with 100, 200 and 300 nodes.

For each of these we examined, for each of 11 re-wiring probabilities:

- 50 networks with average degree 4;
- 50 networks with average degree 6;
- 50 networks with average degree 8.
We created small world networks with 100, 200 and 300 nodes. For each of these we examined, for each of 11 re-wiring probabilities:

- 50 networks with average degree 4;
- 50 networks with average degree 6;
- 50 networks with average degree 8.

As re-wiring probabilities we used:
0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.
We created small world networks with 100, 200 and 300 nodes. For each of these we examined, for each of 11 re-wiring probabilities:

- 50 networks with average degree 4;
- 50 networks with average degree 6;
- 50 networks with average degree 8.

As re-wiring probabilities we used: 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.

This gave a total of 4,950 different networks.
For each of these 4,950 networks we used 600 random initializations.
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These initializations were made by creating 50 random colorings, for each of 12 fixed fractions of $P$ nodes.
For each of these 4,950 networks we used 600 random initializations.

These initializations were made by creating 50 random colorings, for each of 12 fixed fractions of $P$ nodes.

We used the following 12 fractions:
0.01, 0.02, 0.05, 0.10, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50.
For each of these 4,950 networks we used 600 random initializations.

These initializations were made by creating 50 random colorings, for each of 12 fixed fractions of $P$ nodes.

We used the following 12 fractions:
0.01, 0.02, 0.05, 0.10, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50.

This gave 2,970,000 different initializations.
For each of 2,970,000 initializations we ran 50 simulations.
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This gave a total of 148,500,000 simulations.
For each of 2,970,000 initializations we ran 50 simulations.

This gave a total of 148,500,000 simulations.

Each simulation ran until all nodes were of the same type.
Network Simulations

For each of 2,970,000 initializations we ran 50 simulations.
This gave a total of 148,500,000 simulations.
Each simulation ran until all nodes were of the same type.
At each stage all nodes were randomly paired to other nodes.
Network Simulations

For each of 2,970,000 initializations we ran 50 simulations.

This gave a total of 148,500,000 simulations.

Each simulation ran until all nodes were of the same type.

At each stage all nodes were randomly paired to other nodes.

Each node played the strategy that did best among its neighbors (each node is one of its own neighbors).
Number of Initializations for $P$ Wins Proportions for SWN

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Second Dutch-Chinese Seminar on Game Theory and Applications, University of Twente, August 27-29, 2013
Small World Regression Analysis on *Payoff Dominant Wins*

For the Small World Networks Examined:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coef.</th>
<th>Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size</td>
<td>0.000035</td>
<td>positive</td>
</tr>
<tr>
<td>Degree: mean</td>
<td>0.029864</td>
<td>positive</td>
</tr>
<tr>
<td>Share of $P$ nodes</td>
<td>2.454443</td>
<td>positive</td>
</tr>
<tr>
<td>Degree of $P$ nodes: stdev</td>
<td>0.016762</td>
<td>positive</td>
</tr>
<tr>
<td>Power of $P$ nodes: stdev</td>
<td>-6.000691</td>
<td>negative</td>
</tr>
<tr>
<td>Segregation (norm.) of $P$ nodes</td>
<td>-0.017797</td>
<td>negative</td>
</tr>
<tr>
<td>Segregation (norm.) of $R$ nodes</td>
<td>-1.027209</td>
<td>negative</td>
</tr>
<tr>
<td>Constant</td>
<td>0.891018</td>
<td>—</td>
</tr>
</tbody>
</table>

Number of obs. 2,970,000  
R-squared 0.8419
Number of Initializations for Convergence Time for SWN

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For the Small World Networks Examined:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coef.</th>
<th>Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size</td>
<td>0.00381</td>
<td>positive</td>
</tr>
<tr>
<td>Degree: mean</td>
<td>-0.34104</td>
<td>negative</td>
</tr>
<tr>
<td>Share of $P$ nodes</td>
<td>-0.22817</td>
<td>negative</td>
</tr>
<tr>
<td>Degree of $P$ nodes: stdev</td>
<td>-0.95482</td>
<td>negative</td>
</tr>
<tr>
<td>Power of $P$ nodes: stdev</td>
<td>-51.25889</td>
<td>negative</td>
</tr>
<tr>
<td>Segregation (norm.) of $P$ nodes</td>
<td>-0.95487</td>
<td>negative</td>
</tr>
<tr>
<td>Segregation (norm.) of $R$ nodes</td>
<td>-2.58388</td>
<td>negative</td>
</tr>
<tr>
<td>Constant</td>
<td>10.13382</td>
<td>—</td>
</tr>
<tr>
<td>Number of obs.</td>
<td>2,970,000</td>
<td></td>
</tr>
<tr>
<td>R-squared</td>
<td>0.3658</td>
<td></td>
</tr>
</tbody>
</table>
## Small World Classification Tree Analysis

For the Small Data Set of Small World Networks Examined:

<table>
<thead>
<tr>
<th>Selection</th>
<th>Convergence to $P$</th>
<th>Number of Initializations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean</td>
<td>std</td>
</tr>
<tr>
<td>Original dataset</td>
<td>58.3%</td>
<td>44.3%</td>
</tr>
<tr>
<td>Segregation (norm.) of $P$ nodes $&lt; 1.208$</td>
<td>88.2%</td>
<td>23.4%</td>
</tr>
<tr>
<td>Segregation (norm.) of $P$ nodes $\geq 1.208$</td>
<td>9.1%</td>
<td>19.9%</td>
</tr>
</tbody>
</table>

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Comparison of Results for Scale-Free and Small World Networks

1. In both cases *Size, Mean degree, Share of P nodes* and *SD of P degree* have a positive effect on efficient coordination.

2. In both cases *SD of power of P nodes, Segregation of P nodes* and *Segregation of R nodes* have a negative effect on efficient coordination.

3. In both cases *Segregation of P nodes* is the most important variable to decide on convergence to *P* or to *R*.

4. Results differ for speed of convergence in general, but they are very similar when looking only at those initializations with at least 0.75 convergence: only size and segregation norms increase time to convergence.
A Scale Free Network with 3 Types

Initial distr.
(0.4; 0.2; 0.2)

Average Degree 4

<table>
<thead>
<tr>
<th></th>
<th>P</th>
<th>R</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>6,6</td>
<td>0,3</td>
<td>0,1</td>
</tr>
<tr>
<td>R</td>
<td>3,0</td>
<td>4,4</td>
<td>1,2</td>
</tr>
<tr>
<td>S</td>
<td>1,0</td>
<td>2,1</td>
<td>3,3</td>
</tr>
</tbody>
</table>
A Scale Free Network with 3 Types

Initial distr. (0.4; 0.2; 0.2)

Average Degree 4
A Scale Free Network with 3 Types

Initial distr.  
(0.4; 0.2; 0.2)

Average Degree 4

<table>
<thead>
<tr>
<th>P</th>
<th>R</th>
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</tr>
</thead>
<tbody>
<tr>
<td>6,6</td>
<td>0,3</td>
<td>0,1</td>
</tr>
<tr>
<td>3,0</td>
<td>4,4</td>
<td>1,2</td>
</tr>
<tr>
<td>1,0</td>
<td>2,1</td>
<td>3,3</td>
</tr>
</tbody>
</table>
Introduction

A Simulation Study

Characteristics

Results

Other Networks

More Types

A Small World Network with 3 Types

Initial distr. (0.4; 0.2; 0.2)

Re-wiring prob. 0.2

Average Degree 4
A Small World Network with 3 Types

<table>
<thead>
<tr>
<th>P</th>
<th>R</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>6,6</td>
<td>0,3</td>
<td>0,1</td>
</tr>
<tr>
<td>3,0</td>
<td>4,4</td>
<td>1,2</td>
</tr>
<tr>
<td>1,0</td>
<td>2,1</td>
<td>3,3</td>
</tr>
</tbody>
</table>

Initial distr. (0.4; 0.2; 0.2)

Re-wiring prob. 0.2

Average Degree 4
A Small World Network with 3 Types

Initial distr. (0.4; 0.2; 0.2)

Re-wiring prob.
0.2

Average Degree 4

\[
\begin{array}{ccc}
P & R & S \\
6,6 & 0,3 & 0,1 \\
3,0 & 4,4 & 1,2 \\
1,0 & 2,1 & 3,3 \\
\end{array}
\]
A Small World Network with 3 Types

<table>
<thead>
<tr>
<th>P</th>
<th>R</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

Initial distr. (0.4; 0.2; 0.2)

Re-wiring prob. 0.2

Average Degree 4
A Small World Network with 3 Types

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Six Runs in Parallel

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Six Runs in Parallel
Six Runs in Parallel

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Second Dutch-Chinese Seminar on Game Theory and Applications, University of Twente, August 27-29, 2013
Six Runs in Parallel
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Second Dutch-Chinese Seminar on Game Theory and Applications, University of Twente, August 27-29, 2013
Six Runs in Parallel
Thank you for your attention! Comments will be appreciated!

Presentation and paper will soon be available at https://dke.maastrichtuniversity.nl/f.thuijsman/