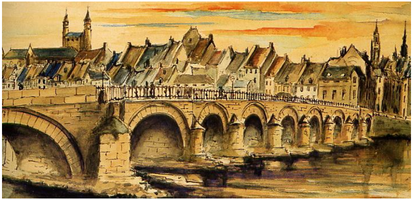
Network Characteristics and Efficient Coordination



Frank Thuijsman

joint work with

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Maastricht University

Outline

Coordination Game

	P	R
P	a, a	b, c
R	c, b	d, d

Assumptions:

1. $a > c, d > b$: pure equilibria (P, P) and (R, R);
2. $a > d$: payoff on P Pareto dominates payoff on R;
3. $c > b$: in case of mis-coordination, R is safer.

Population of Players

Assumptions:

1. even number of players;
2. players are connected in (social) network;
3. at discrete stages $1, 2, 3, \ldots$ players are randomly matched to other players;
4. at each stage each player chooses P or R by imitating neighbor with highest realized payoff;
5. neighbors include self.

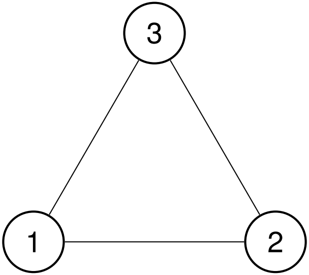
Goal of Study

We want to investigate the influence of network characteristics:

1. on convergence to the efficient outcome P;
2. on the speed of convergence to a homogeneous population.

Scale-Free Networks

Method of construction



Motivation:
Scale-free networks match empirical data on networks
Few nodes with high degree, many nodes with low degree.

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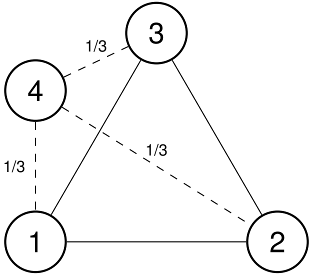

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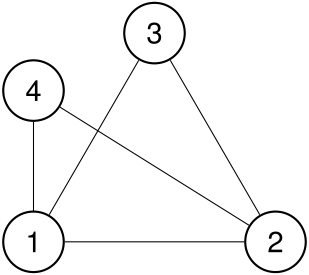

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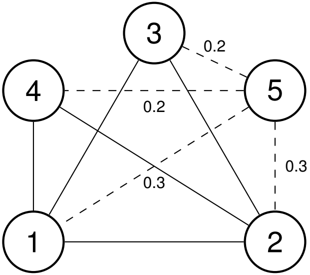

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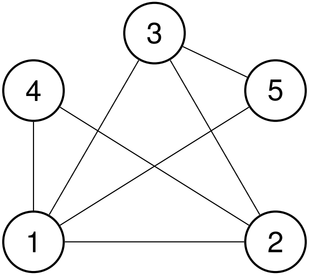

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This gave a total of 900 different networks.

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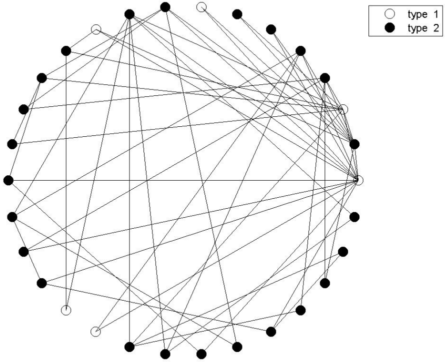
An Example on a Scale Free Network



	P	R
P	$6,6$	$0,3$
R	$3,0$	$4,4$

Initially 20% P, type 1, white

Average Degree 4

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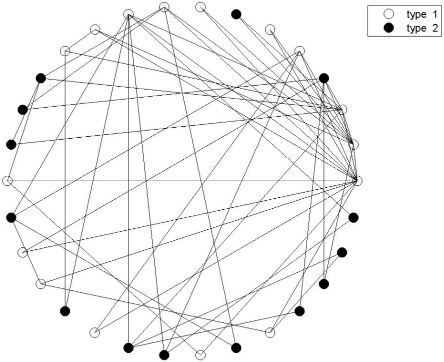

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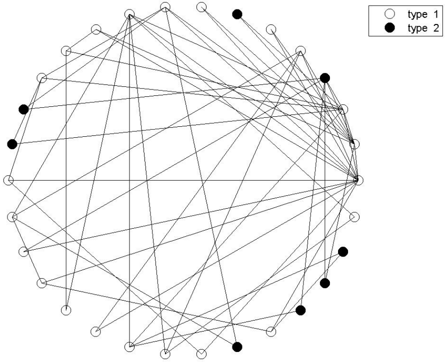

	P	R
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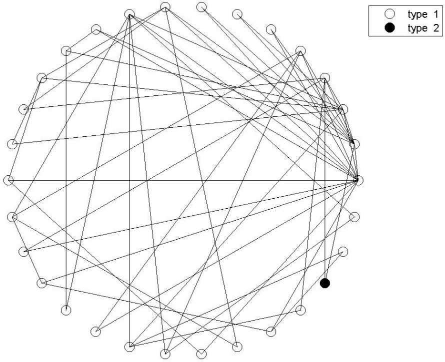

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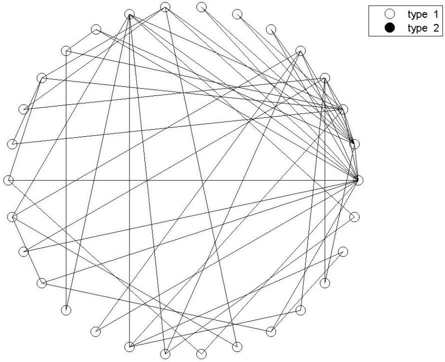

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Network Specific Characteristics (NSC)

- *Size*:
- *Density*:
- *Degree*:
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Initial Assignment of Strategies (IAS)

- *Share of P nodes*: fraction of P nodes
- *Degree of P nodes*: mean and s.d. of degree per P node
- *Power of P nodes*: sum, mean and s.d.
- *Segregation of P nodes*: measure using random walks
- *Segregation of R nodes*: same

Variables to Explain

- *Payoff Dominant Wins*:
- *Mean Convergence Time*:

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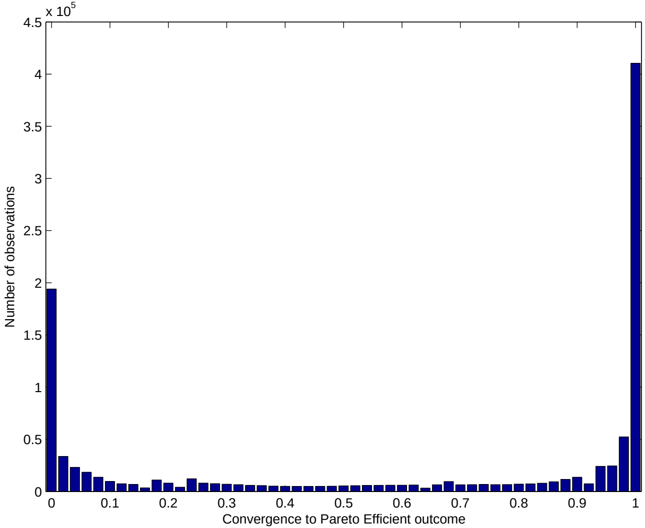
Each of these is measured over 100 runs for any specific choice of initialized network.

Number of Initializations for P Wins Proportions

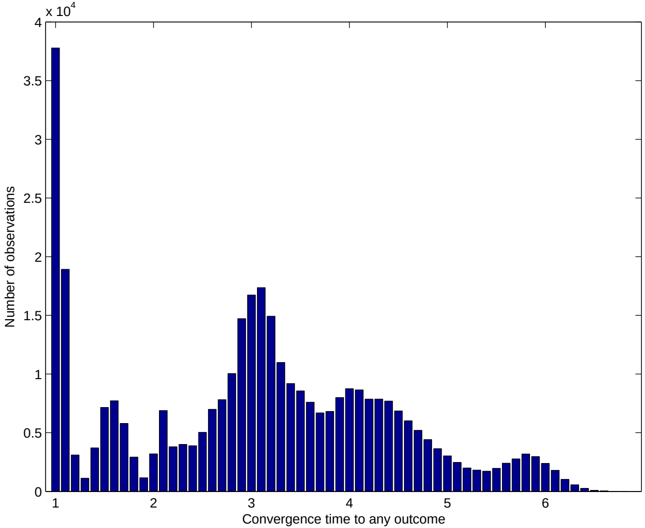


Regression Analysis on *Payoff Dominant Wins*

For the Scale Free Networks Examined:

Variable	Coef.	Effect
Size	0.000117	positive
Degree: mean	0.011441	positive
Share of P nodes	2.182151	positive
Degree of P nodes: stdev	0.014224	positive
Power of P nodes: stdev	-2.428675	negative
Segregation (norm.) of P nodes	-0.053563	negative
Segregation (norm.) of R nodes	-0.134324	negative
Constant	0.171971	—
Number of obs.	1,080,000	
R-squared	0.8478	

Number of Initializations for *Convergence Time*



Regression Analysis on *Convergence Time*

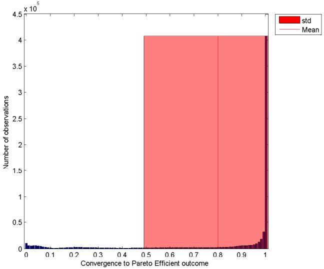
For the Scale Free Networks Examined:

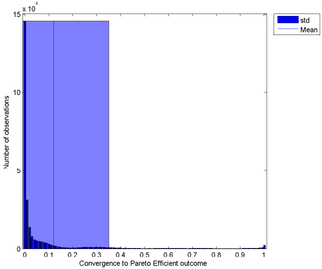
Variable	Coef.	Effect
Size	0.00233	positive
Degree: mean	-0.37935	negative
Share of *P* nodes	0.30642	positive
Degree of *P* nodes: stdev	0.10754	positive
Power of *P* nodes: stdev	4.10292	positive
Segregation (norm.) of *P* nodes	-0.81094	negative
Segregation (norm.) of *R* nodes	1.66375	positive
Constant	3.64648	—
Number of obs.	1,080,000	
R-squared	0.4691	

Classification Tree Analysis on *Payoff Dominant Wins*

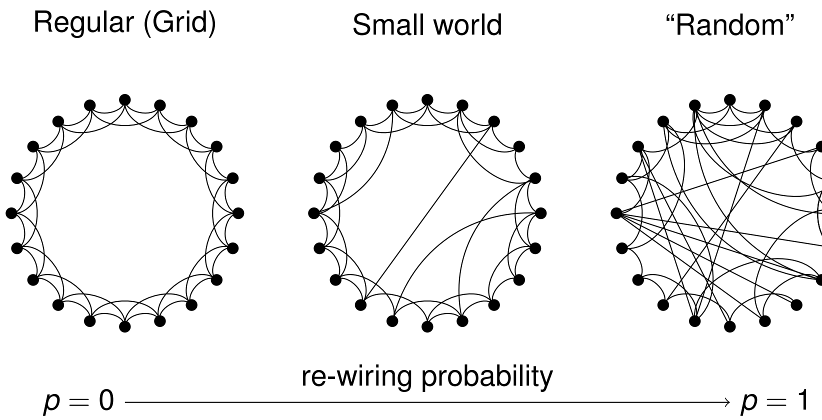
For the Scale Free Networks Examined:

Selection	Convergence to P mean	std	Number of Initializations
Original dataset	63.2%	42.0%	(1,050,000)
Segregation (norm.) of P nodes < 1.302	80.1%	31.7%	(788,193)
Segregation (norm.) of P nodes ≥ 1.302	12.0%	23.7%	(261,807)





Small World Networks



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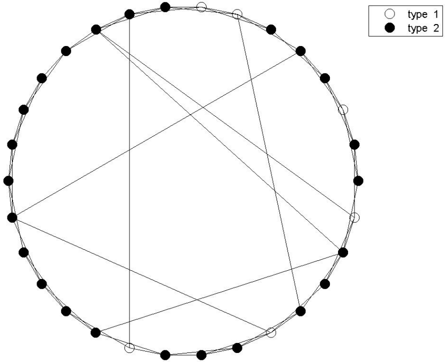

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P	6, 6	0, 3
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Re-wiring prob. 0.2

Average Degree 4

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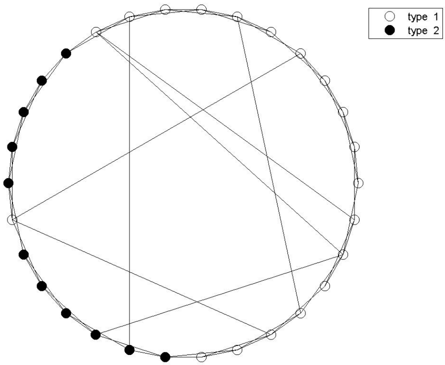

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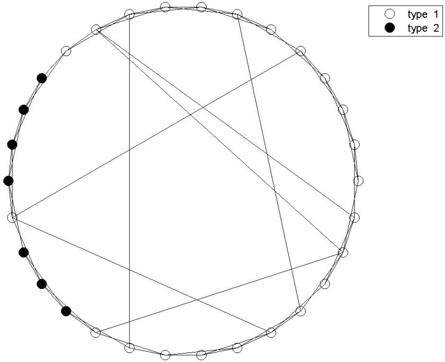

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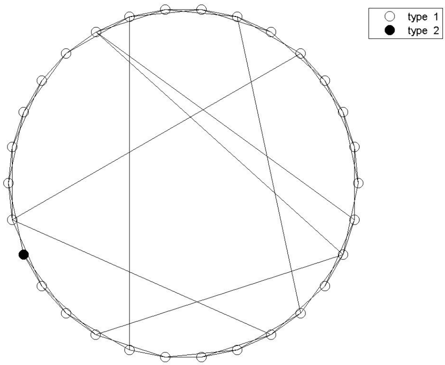

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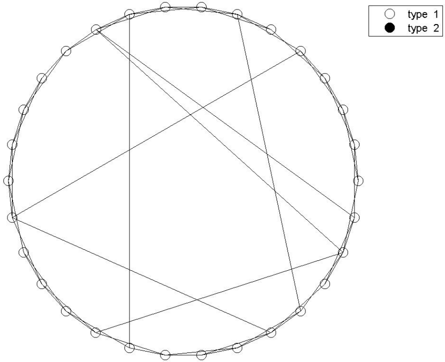

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For each of these we examined, for each of 11 re-wiring probabilities:

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This gave a total of 4,950 different networks.

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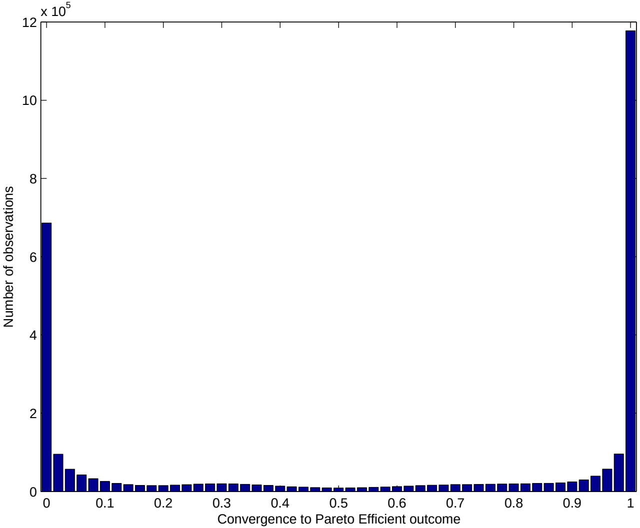
Number of Initializations for P Wins Proportions for SWN

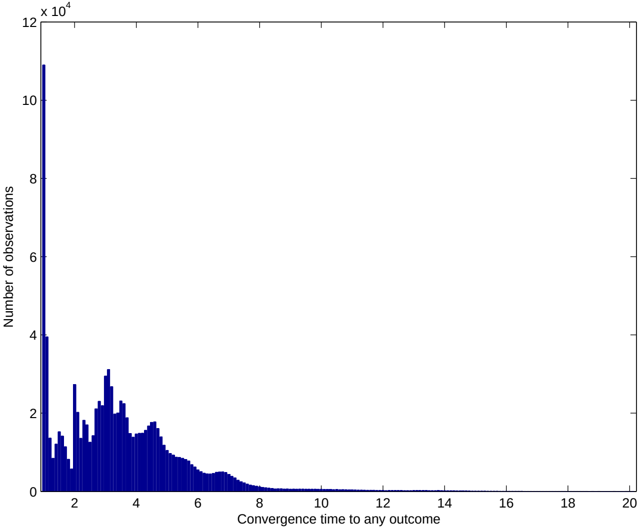


Small World Regression Analysis on *Payoff Dominant Wins*

For the Small World Networks Examined:

Variable	Coef.	Effect
Size	0.000035	positive
Degree: mean	0.029864	positive
Share of *P* nodes	2.454443	positive
Degree of *P* nodes: stdev	0.016762	positive
Power of *P* nodes: stdev	-6.000691	negative
Segregation (norm.) of *P* nodes	-0.017797	negative
Segregation (norm.) of *R* nodes	-1.027209	negative
Constant	0.891018	—
Number of obs.	2,970,000	
R-squared	0.8419	

Number of Initializations for *Convergence Time* for SWN



Small World Regression Analysis on *Convergence Time*

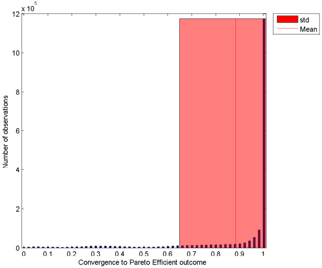
For the Small World Networks Examined:

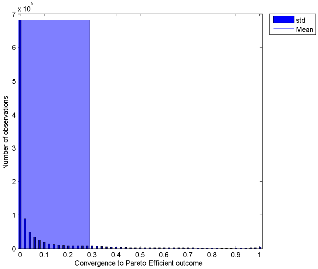
Variable	Coef.	Effect
Size	0.00381	positive
Degree: mean	-0.34104	negative
Share of P nodes	-0.22817	negative
Degree of P nodes: stdev	-0.95482	negative
Power of P nodes: stdev	-51.25889	negative
Segregation (norm.) of P nodes	-0.95487	negative
Segregation (norm.) of R nodes	-2.58388	negative
Constant	10.13382	—
Number of obs.	2,970,000	
R-squared	0.3658	

Small World Classification Tree Analysis

For the Small Data Set of Small World Networks Examined:

Selection	Convergence to P mean	std	Number of Initializations
Original dataset	58.3%	44.3%	(2,970,000)
Segregation (norm.) of P nodes < 1.208	88.2%	23.4%	(1,845,824)
Segregation (norm.) of P nodes ≥ 1.208	9.1%	19.9%	(1,124,176)

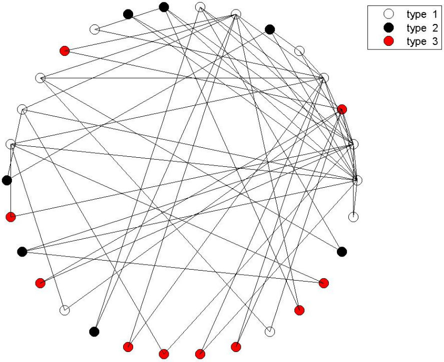




Comparison of Results for Scale-Free and Small World Networks

1. In both cases *Size*, *Mean degree*, *Share of P nodes* and *SD of P degree* have a positive effect on efficient coordination.
2. In both cases *SD of power of P nodes*, *Segregation of P nodes* and *Segregation of R nodes* have a negative effect on efficient coordination.
3. In both cases *Segregation of P nodes* is the most important variable to decide on convergence to P or to R.
4. Results differ for speed of convergence in general, but they are very similar when looking only at those initializations with at least 0.75 convergence: only size and segregation norms increase time to convergence.

A Scale Free Network with 3 Types



	P	R	S
P	6, 6	0, 3	0, 1
R	3, 0	4, 4	1, 2
S	1, 0	2, 1	3, 3

Initial distr. (0.4; 0.2; 0.2)

Average Degree 4

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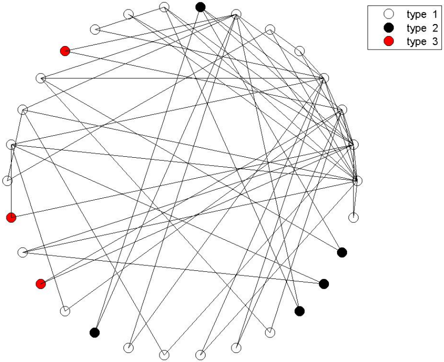

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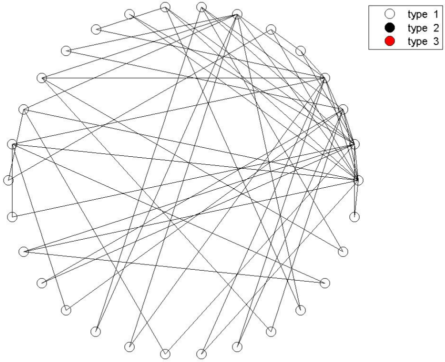

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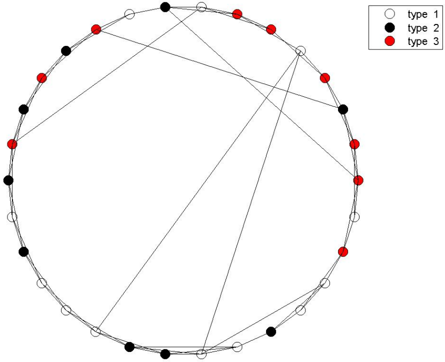

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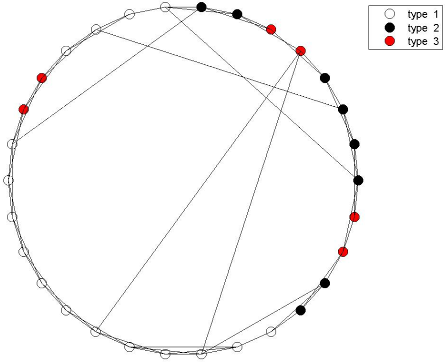
Initial distr.
(0.4; 0.2; 0.2)

Re-wiring prob.
0.2

Average Degree 4

A Small World Network with 3 Types



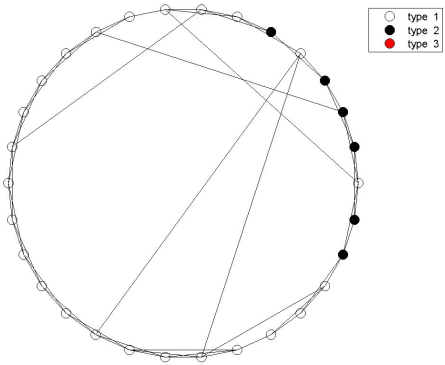
	P	R	S
P	6, 6	0, 3	0, 1
R	3, 0	4, 4	1, 2
S	1, 0	2, 1	3, 3

Initial distr.
(0.4; 0.2; 0.2)

Re-wiring prob.
0.2

Average Degree 4

A Small World Network with 3 Types



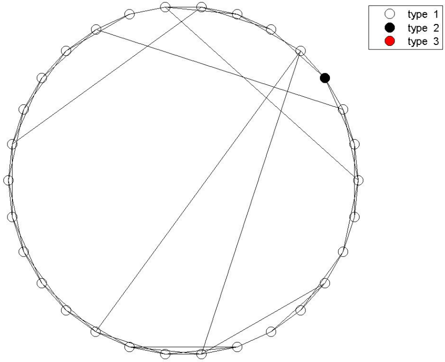
	P	R	S
P	6, 6	0, 3	0, 1
R	3, 0	4, 4	1, 2
S	1, 0	2, 1	3, 3

Initial distr.
(0.4; 0.2; 0.2)

Re-wiring prob.
0.2

Average Degree 4

A Small World Network with 3 Types



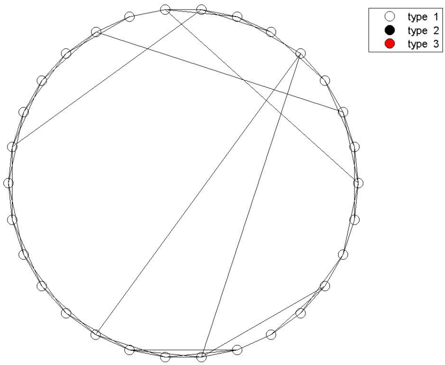
	P	R	S
P	$6,6$	$0,3$	$0,1$
R	$3,0$	$4,4$	$1,2$
S	$1,0$	$2,1$	$3,3$

Initial distr.
$(0.4; 0.2; 0.2)$

Re-wiring prob.
0.2

Average Degree 4

A Small World Network with 3 Types



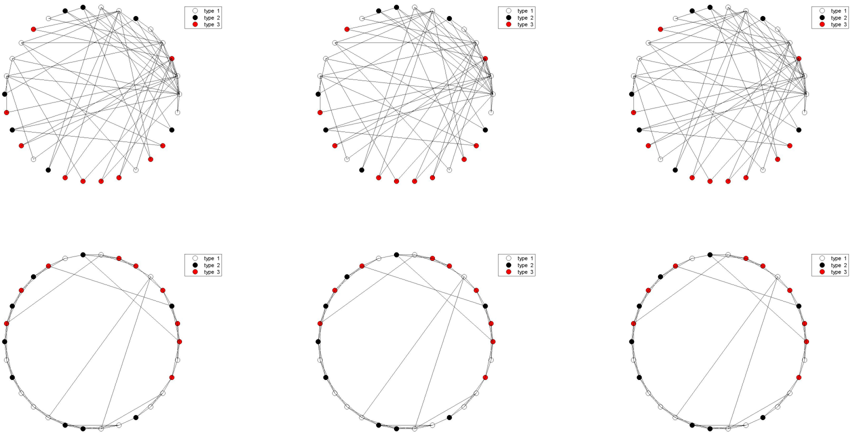
	P	R	S
P	6, 6	0, 3	0, 1
R	3, 0	4, 4	1, 2
S	1, 0	2, 1	3, 3

Initial distr.
(0.4; 0.2; 0.2)

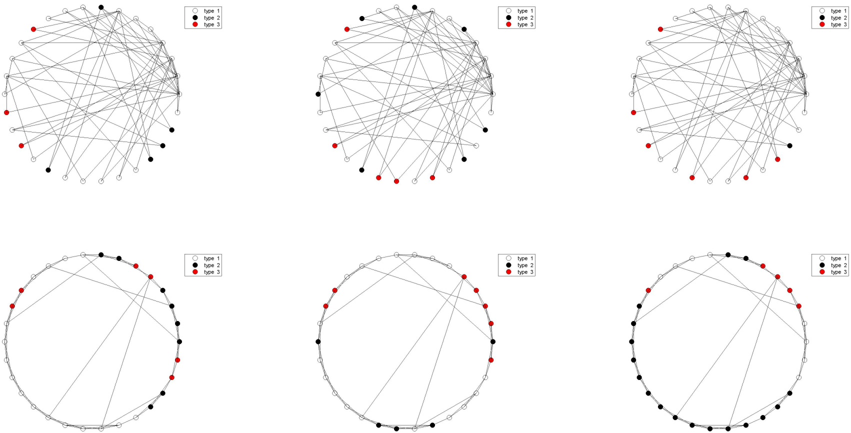
Re-wiring prob.
0.2

Average Degree 4

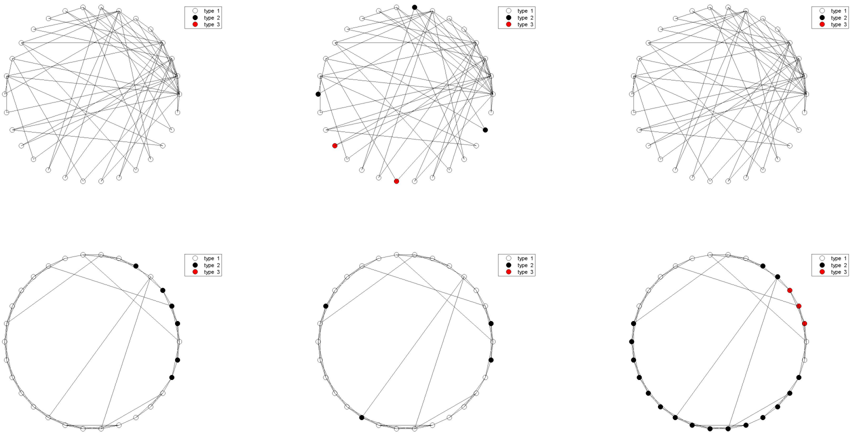
Six Runs in Parallel



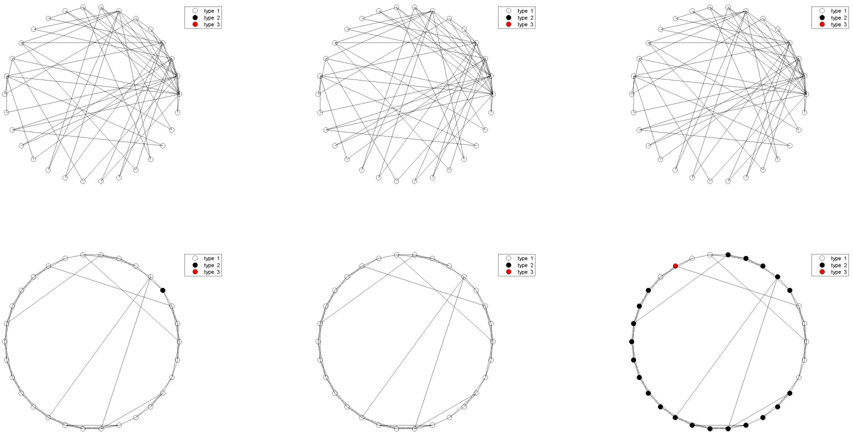
Six Runs in Parallel



Six Runs in Parallel



Six Runs in Parallel



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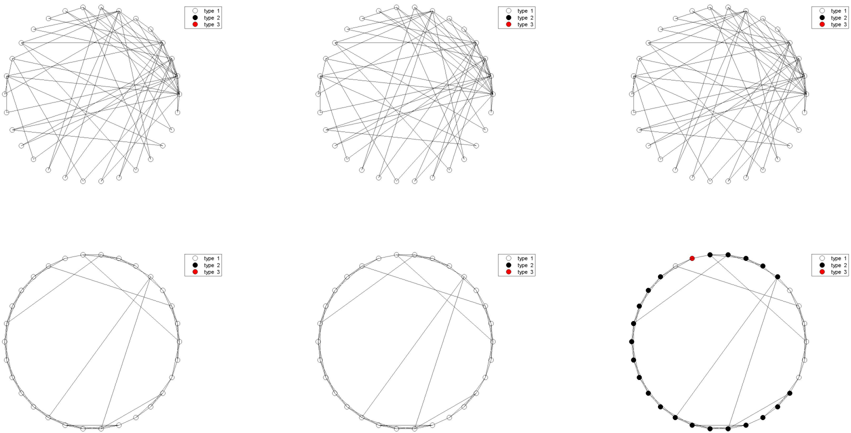

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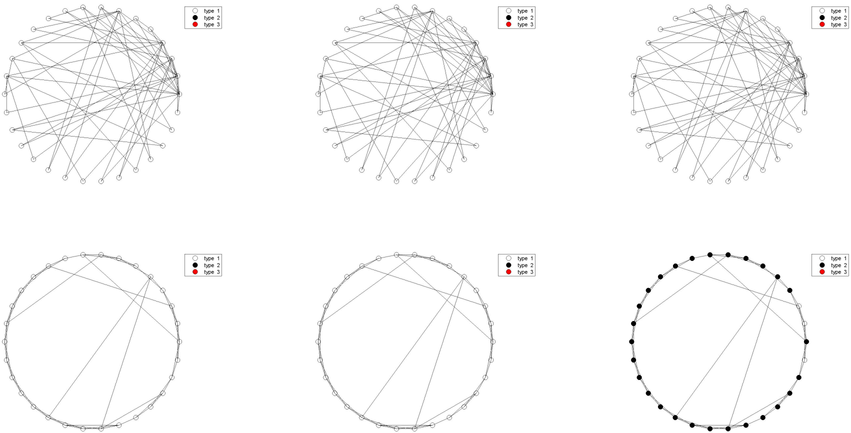
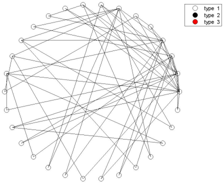
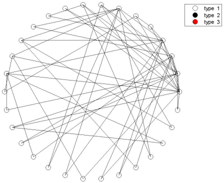
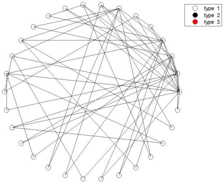
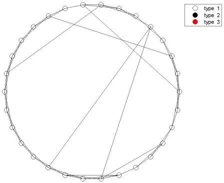


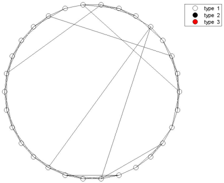
Six Runs in Parallel

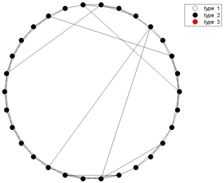




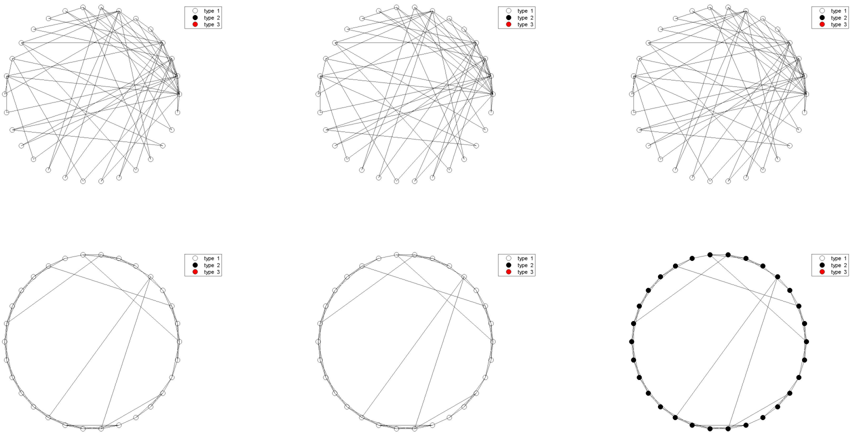








Six Runs in Parallel



Thanks

Thank you for your attention!
Comments will be appreciated!

Presentation and paper will soon be available at
https://dke.maastrichtuniversity.nl/f.thuijsman/