Evolutionary Games and Periodic Fitness

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Abstract
One thing that nearly all stability concepts in Evolutionary Game Theory [1] have in common is that they use a time-independent fitness matrix. Although this is a reasonable assumption for mathematical purposes, in many situations in real life it seems to be too restrictive. We present a model of an evolutionary game, driven by replicator dynamics [2], where the fitness matrix is a variable rather than a constant, i.e. the fitness matrix is time-dependent. In particular, by considering periodically changing fitness matrices, we model seasonal effects in evolutionary games. We discuss a model with a continuously changing fitness matrix as well as a related model in which the changes occur periodically at discrete points in time. A numerical analysis shows stability of the periodic orbits that are observed. Moreover, trajectories leading to these orbits from arbitrary starting points synchronize their motion in time.

Periodic Fitness
We investigate the replicator dynamics and a type of stability of the population distribution in evolutionary games with a time-dependent, periodic fitness matrix $A(t)$. We will call this periodic stability.

Time-dependent replicator-dynamics:
$$\dot{x}_i = x_i (e_i A(t)x^T - x A(t)x^T) \quad \text{for} \quad i=1,\ldots,n, \quad t \geq 0$$

Main Example
The main example is based on the idea of having three types, two of which have a fitness that periodically depends on time, sometimes doing good, sometimes bad, while for another type the fitness is not affected by time.

$$A(t) = \begin{cases} 0 & 0 & 2 + \sigma \cos(\rho t) \\ \alpha & 0 & \alpha \\ 2 - \sigma \cos(\rho t) & 0 & 0 \end{cases}$$

Here:
- $\sigma$: size/amplitude of the variation
- $\rho$: time to complete one period
- $\alpha$: fitness of the unaffected individuals

The pictures below correspond to $\sigma = 0.88$, $\rho = 0.1$, $\alpha = 1$

Conclusion
We have discovered that it is possible to reach a periodically stable population pattern when following the replicator dynamics based on a periodically changing fitness matrix. Because the fitness matrix changes continuously in time, the force field of the dynamics that makes the population move into the direction of a particular population distribution, changes with it.

In our stepwise periodic model, we have sharp changes of the fitness matrix at fixed times, and the number of attraction points is finite. In fact, for the particular case examined, there is just one attraction point all the time, but the combined process never reaches it.

We also observed that processes starting at different points of the simplex very quickly start following synchronized patterns.

We want to remark that we could only employ the Ariadne [3] tool for showing stability of the orbits observed, because the fitness matrix was changing in a deterministic way. Such would not have been possible when it was changing in a stochastic way, because then there would not necessarily be any closed loop.

A challenging question is to predict the existence of periodically stable orbits based on the periodic fitness matrix. A second question is to find closed form expressions for the periodic orbits observed.

References

Movies
Movies for the dynamic processes discussed can be observed at http://www.youtube.com/watch?v=C65Z7fcLA4s or using the keyword: Periodically stable orbits