Evolutionary Game Theory and Local Interactions

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Abstract
In the classical approach of evolutionary game theory, the so called replicator dynamics [1] are the driving forces behind the development of a population in time. In such models the population usually consists of a continuum of individuals that each belong to a finite number of types. Individuals meet randomly and these interactions have an impact on the fitness of their types. The fitness payoffs are given in a fitness matrix. Types that are doing better than average increase in number, while those that are doing worse decrease. The fact that individuals meet randomly implies that the location of these individuals plays no role. In this research we examine what happens when individuals do have a specific location and only interact with their local neighborhood.

Computation of payoff for the cells
Consider a cell $c$ 
The neighbors of $c$ are $N_c$ 
The fitness of the individuals in cell $c$ is $f(c) = \sum_{d \in N_c} A(c,d)$ 

Where $A(c,d)$ is the fitness payoff for the type of cell $c$ when it interacts with the type of cell $d$, thus we calculate the average payoff of individuals playing against their neighbors.

Neighbor distribution of $a$ is $n_a = \left( \begin{array}{l} 2 \cr 2 \cr 2 \end{array} \right)$

Neighbor distribution of $d$ is $n_d = \left( \begin{array}{l} 2 \cr 2 \cr 2 \end{array} \right)$

$f(a) = c_2 A_{a,d} = 14$

$f(d) = c_2 A_{c,a} = 14$

This calculation is made for all cells in the field simultaneously. Then we look for each cell which type has the highest fitness of its neighborhood.

Predator-prey behavior using expected local fitness

Extensions of the model
- Extended neighborhoods
- Weighted influence of these neighborhoods
- Directional neighborhoods
- Different fitness matrices depending on the location
- Fitness matrix changing in time
- Multiple individuals per cell
- Mutation/death of individuals
- Resource depletion

References

Applicatons using realized local fitness
Adapting the settings of our model, we can reproduce the observations made in the article by B.Kerr, M.Riley, M.Feldman et al. [2] named “Local dispersal promotes biodiversity in a real-life game of rock-paper-scissors”.

In this article toxic (C) cells are put together with resistant (R) and sensitive (S) cells. The properties of these three types of cells are that R is able to resist to the toxicity of C and thus performs better, C kills S with its poison, and S beats R by reproducing faster.

Unlike what is claimed, the underlying payoff matrix is not the one of a rock-paper-scissors game because in the experiments, when looking at the global interaction neighborhood, the resistant type clearly dominates the process as one can see in the figures below. These pairwise relationships can be represented by the following fitness matrix:

\[
\begin{pmatrix}
S & C & R \\
C & 1 & 0 \\
R & 1 & 1 \\
\end{pmatrix}
\]

In this matrix C beats S with a payoff of 1, R beats C with a payoff of 1, and S beats R with a payoff of 0 (against a payoff of -1 for R). However the (global) replicator dynamics will drive the process towards a population consisting almost exclusively of Resistant strains.

Using a process of local pairwise interactions, i.e. using realized local fitness in which two individuals are randomly selected and the one having the highest payoff wins, our model exhibits similar observations as those explained in the papers mentioned.

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Movies
Movies for the processes discussed can be observed at https://www.youtube.com/watch?v=GMF03EM1tcY
or using the keyword: Local Replicator Dynamics