



Maastricht University

Department of Knowledge Engineering

Mathe-Dual Society, RWTH Aachen

Science and the Stability of Marriage

Frank Thuijsman

April 9, 2013



Outline

1. 2-Person Zero-Sum Games: John von Neumann
2. A Solution for n -Person Games: John Nash
3. Stability in Marriages: Lloyd Shapley
4. Stability among Widows: Bob Aumann

2-Person Zero-Sum Games



1	3
4	2

1928

John von Neumann

2-Person Zero-Sum Games



$1 - p$	1	3
p	4	2

John von Neumann

2-Person Zero-Sum Games



$1 - p$	1	3
p	4	2
	$1 + 3p$	$3 - p$

John von Neumann

2-Person Zero-Sum Games



$1 - p$	1	3
p	4	2

$$1 + 3p$$

$$3 - p$$

by $p = 1/2$
player 1 gets at least 2.5

John von Neumann

2-Person Zero-Sum Games



$1 - q$	q	
1	3	$1 + 2q$
4	2	$4 - 2q$

by $q = 3/4$
player 1 gets at most 2.5

John von Neumann

2-Person Zero-Sum Games



$1 - q$	q	
1	3	$1 + 2q$
4	2	$4 - 2q$

by $q = 3/4$
player 1 gets at most 2.5

2.5 is the value of the game

John von Neumann

2-Person Zero-Sum Games



The MiniMax Theorem

Every finite matrix A has a value, i.e.

$$\max_x \min_y xAy^T = \min_y \max_x xAy^T$$

1928

John von Neumann

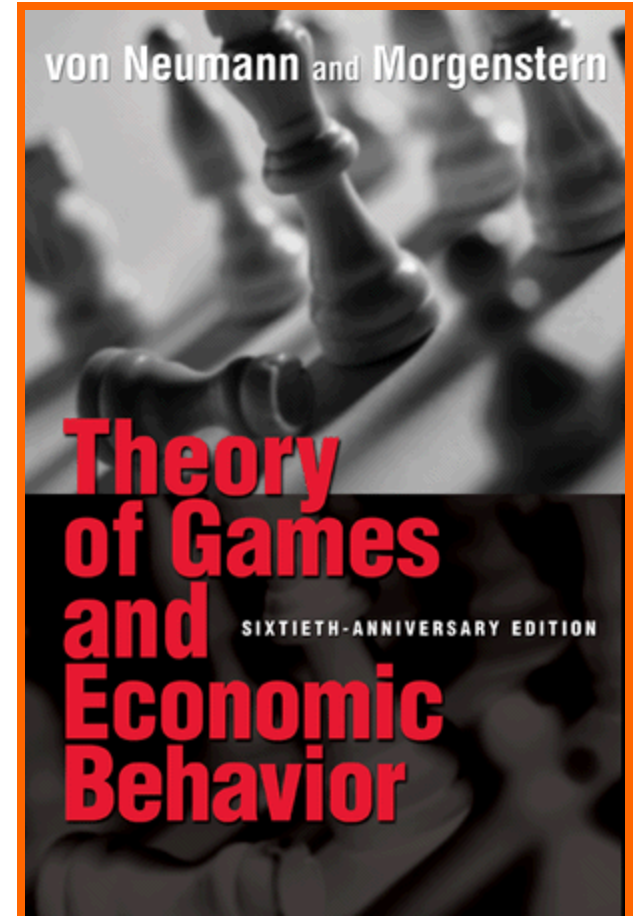
Launching the Field of Game Theory



John von Neumann



Oskar Morgenstern



1944

From 2-Person Zero-Sum Games ...



1	3
4	2

1928

John von Neumann

... to a Solution for n -Person Games



John von Neumann

1,6	3,-4
4,0	2,5

1951



John Nash

... to a Solution for n -Person Games

1/2	1,6	3,-4
1/2	4,0	2,5

Player 1 would get 2.5 2.5



John Nash

... to a Solution for n -Person Games

1/2	1,6	3,-4
1/2	4,0	2,5

Player 1 would get 2.5 2.5
 Player 2 would get 3 0.5



John Nash

... to a Solution for n -Person Games

1/2	1,6	3,-4
1/2	4,0	2,5

Player 1 would get 2.5 2.5

Player 2 would get 3 0.5

If player 1 does what is best for himself,
then *player 2 would prefer to play Left!*

This does NOT give a stable solution!



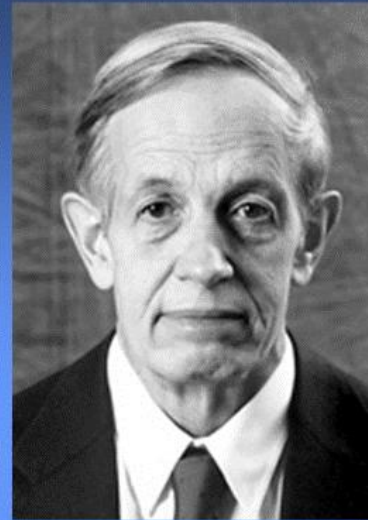
John Nash

Stability in n -Player Games



**The Sveriges Riksbank
Prize in Economic
Sciences in Memory of
Alfred Nobel 1994**

"for their pioneering analysis of
equilibria in the theory of non-
cooperative games"



John F. Nash Jr.
USA

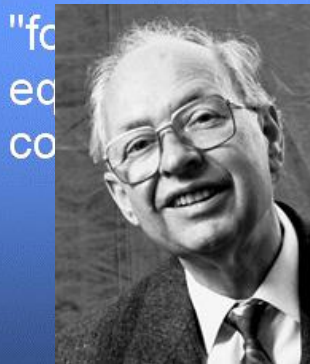
Princeton University
Princeton, NJ, USA
b. 1928

1994

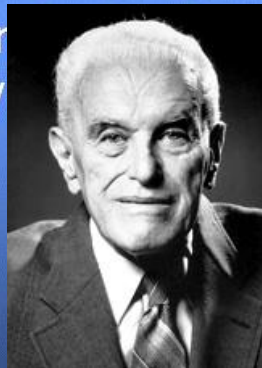
Stability in n -Player Games



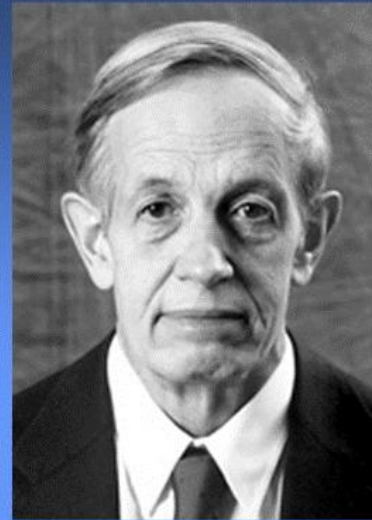
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"for
economic theory
co
games"



Reinhard Selten John Harsanyi



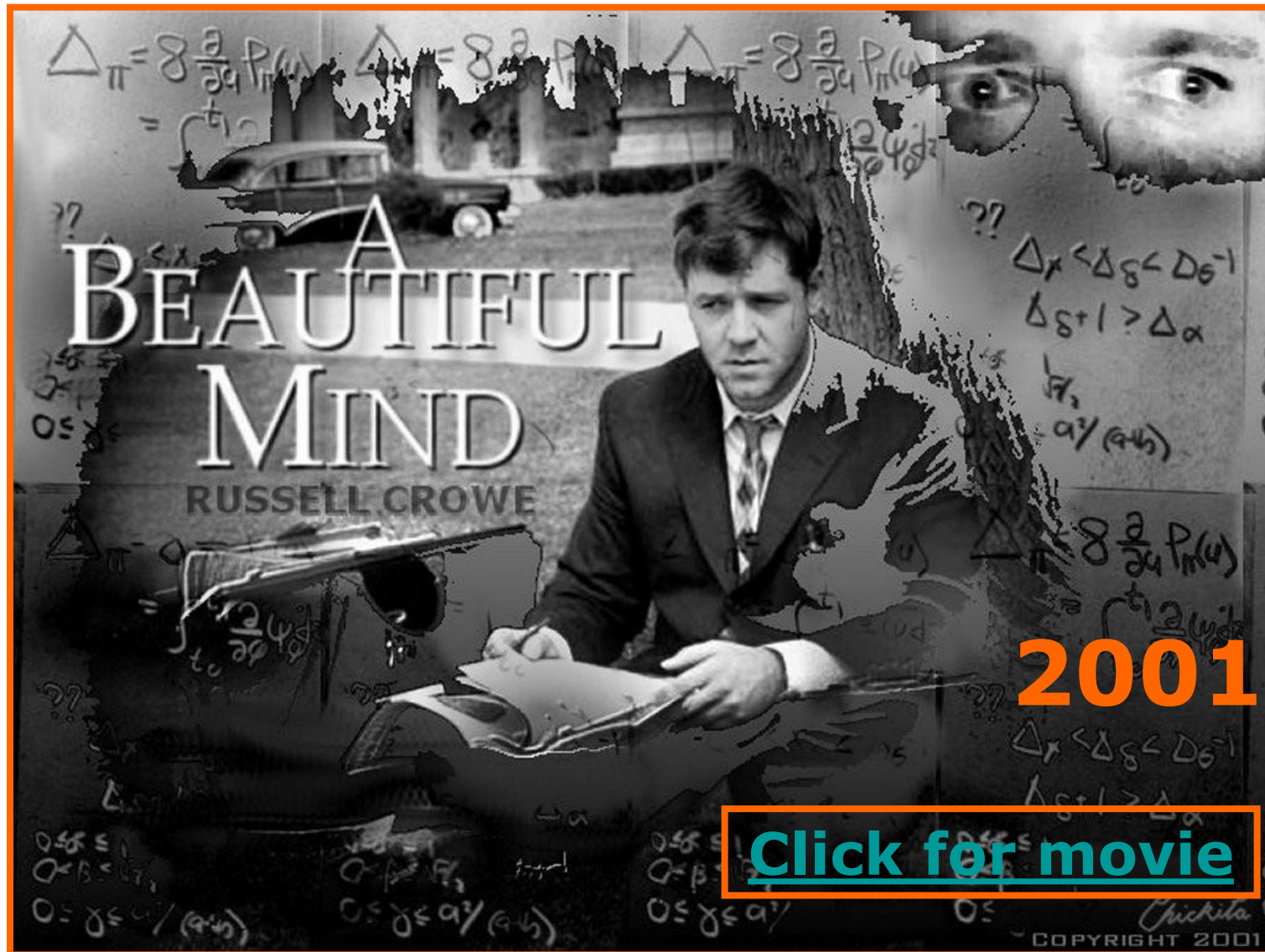
John F. Nash Jr.

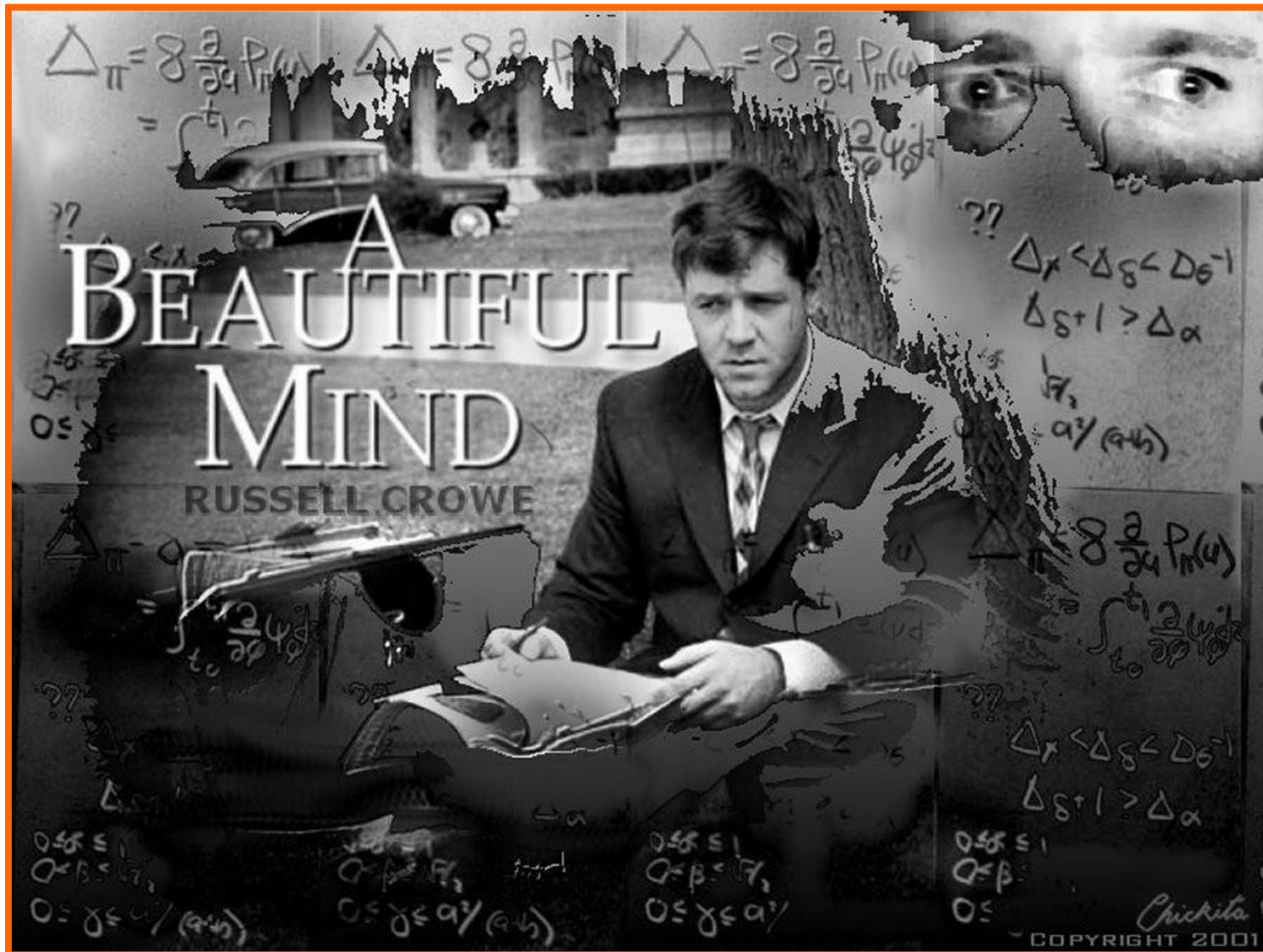
USA

Princeton University
Princeton, NJ, USA

b. 1928

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... NOT everyone for himself !!!

1/2	1,6	3,-4
1/2	4,0	2,5

Player 1 would get 2.5 2.5

Player 2 would get 3 0.5

We have to approach it differently!



John Nash

... NOT everyone for himself !!!

$1 - p$	1, 6	3, -4
p	4, 0	2, 5
	$6 - 6p$	$-4 + 9p$

for $p = 2/3$
player 2 is indifferent
 between Left and Right;
 each gives **2**



John Nash

... NOT everyone for himself !!!

	$1 - q$	q
$1 + 2q$	1,6	3,-4
$4 - 2q$	4,0	2,5

for $q = 3/4$
 player 1 is indifferent
 between Top and Bottom;
 each gives 2.5



John Nash

... NOT everyone for himself !!!

	$1 - q$	q
$1 + 2q$	1, 6	3, -4
$4 - 2q$	4, 0	2, 5

**$((1/3, 2/3), (1/4, 3/4))$
is an equilibrium for this game;
player 1 gets 2.5 and
player 2 gets 2.**



John Nash

... NOT everyone for himself !!!

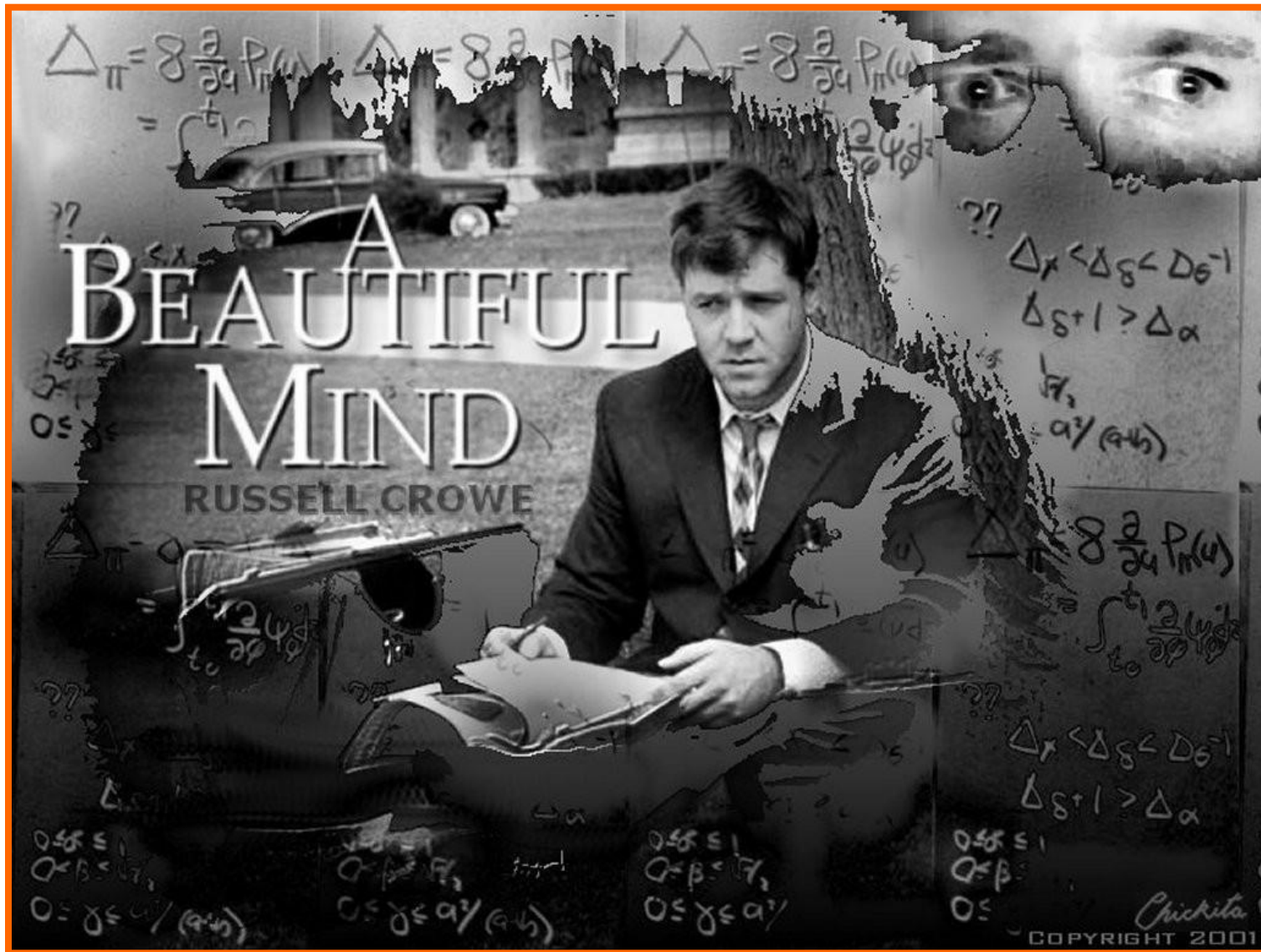
Equilibrium Existence:

Every finite n -person game has at least one equilibrium.

1951



John Nash



Stability in *Marriage*



John Nash and his Wife Alicia



**1957
-1963**

John Nash and his Wife Alicia



**2001
- Today**

Marriage Problems **Today**

Berliner Zeitung

Wirtschaft 15.10.2012

WIRTSCHAFTSNOBELPREIS FÜR US-FORSCHER

Ehrung für die „Spieler“ Roth und Shapley

Von Mario Müller



Das Komitee bei der Bekanntgabe der Gewinner.

Foto: dapd

Der Nobelpreis für Wirtschaft geht an die US-Amerikaner Alvin Roth und Lloyd Shapley, zwei Pioniere der Spieltheorie. Sie haben mit ihren Arbeiten tausenden Ärzten und Schülern geholfen, eine gute Einrichtung zu finden. Auch Organspender profitieren.

An aktuellen Problemen herrscht in Sachen Wirtschaft beileibe kein Mangel. Hier nur eine kleine Auswahl: die Krise der Euro-Zone, das verhängnisvolle Versagen der Finanzmärkte oder die weltweit immer größer werdende Kluft zwischen Arm und Reich. Alles Themen, die von höchster gesellschaftlicher Relevanz sind und auch innerhalb der Ökonomen-Zunft intensiv und kontrovers diskutiert werden.

Marriage Problems **Today**



Marriage Problems: Lloyd Shapley



Marriage Problems: Lloyd Shapley



Marriage Problems

	1	2	3	4	5
Anny	Freddy	Harry	Kenny	Gerry	Lenny
Betty	Gerry	Kenny	Freddy	Harry	Lenny
Conny	Lenny	Harry	Gerry	Freddy	Kenny
Dolly	Harry	Lenny	Freddy	Gerry	Kenny
Emmy	Harry	Kenny	Gerry	Lenny	Freddy

	1	2	3	4	5
Freddy	Conny	Betty	Anny	Emmy	Dolly
Gerry	Dolly	Anny	Betty	Emmy	Conny
Harry	Emmy	Anny	Dolly	Betty	Conny
Kenny	Emmy	Conny	Anny	Dolly	Betty
Lenny	Emmy	Anny	Betty	Conny	Dolly

Marriage Problems

	1	2	3	4	5
Anny	Freddy	Harry	Kenny	Gerry	Lenny
Betty	Gerry	Kenny	Freddy	Harry	Lenny
Conny	Lenny	Harry	Gerry	Freddy	Kenny
Dolly	Harry	Lenny	Freddy	Gerry	Kenny
Emmy	Harry	Kenny	Gerry	Lenny	Freddy

	1	2	3	4	5
Freddy	Conny	Betty	Anny	Emmy	Dolly
Gerry	Dolly	Anny	Betty	Emmy	Conny
Harry	Emmy	Anny	Dolly	Betty	Conny
Kenny	Emmy	Conny	Anny	Dolly	Betty
Lenny	Emmy	Anny	Betty	Conny	Dolly

Marriage Problems

	1	2	3	4	5
Anny	Freddy		Kenny	Gerry	Lenny
Betty	Gerry	Kenny	Freddy		Lenny
Conny	Lenny		Gerry	Freddy	Kenny
Dolly		Lenny	Freddy	Gerry	Kenny
		Kenny	Gerry	Lenny	Freddy

	1	2	3	4	5
Freddy	Conny	Betty	Anny		Dolly
Gerry	Dolly	Anny	Betty		Conny
		Anny	Dolly	Betty	Conny
Kenny		Conny	Anny	Dolly	Betty
Lenny		Anny	Betty	Conny	Dolly

Marriage Problems

	1	2	3	4	5
Anny	Freddy		Kenny	Gerry	Lenny
Betty	Gerry	Kenny	Freddy		Lenny
Conny	Lenny		Gerry	Freddy	Kenny
Dolly		Lenny	Freddy	Gerry	Kenny
		Kenny	Gerry	Lenny	Freddy

	1	2	3	4	5
Freddy	Conny	Betty	Anny		Dolly
Gerry	Dolly	Anny	Betty		Conny
		Anny	Dolly	Betty	Conny
Kenny		Conny	Anny	Dolly	Betty
Lenny		Anny	Betty	Conny	Dolly

Marriage Problems

	1	2	3	4	5
Anny	Freddy		Kenny	Gerry	Lenny
Betty	Gerry	Kenny	Freddy		Lenny
Conny	Lenny		Gerry	Freddy	Kenny
Dolly		Lenny	Freddy	Gerry	Kenny
		Kenny	Gerry	Lenny	Freddy

	1	2	3	4	5
Freddy	Conny	Betty	Anny		Dolly
Gerry	Dolly	Anny	Betty		Conny
		Anny	Dolly	Betty	Conny
Kenny		Conny	Anny	Dolly	Betty
Lenny		Anny	Betty	Conny	Dolly

Marriage Problems

	1	2	3	4	5
Anny	Freddy		Kenny	Gerry	Lenny
Betty	Gerry	Kenny	Freddy		Lenny
Conny	Lenny		Gerry	Freddy	Kenny
Dolly		Lenny	Freddy	Gerry	Kenny
		Kenny	Gerry	Lenny	Freddy

	1	2	3	4	5
Freddy	Conny	Betty	Anny		Dolly
Gerry	Dolly	Anny	Betty		Conny
		Anny	Dolly	Betty	Conny
Kenny		Conny	Anny	Dolly	Betty
Lenny		Anny	Betty	Conny	Dolly

Marriage Problems

	1	2	3	4	5
Anny	Freddy		Kenny	Gerry	Lenny
Betty	Gerry	Kenny	Freddy		Lenny
Conny	Lenny		Gerry	Freddy	Kenny
Dolly		Lenny		Gerry	Kenny
		Kenny	Gerry	Lenny	Freddy

	1	2	3	4	5
Freddy	Conny	Betty	Anny		
Gerry	Dolly	Anny	Betty		Conny
		Anny	Dolly	Betty	Conny
Kenny		Conny	Anny	Dolly	Betty
Lenny		Anny	Betty	Conny	Dolly

Marriage Problems

	1	2	3	4	5
Anny	Freddy		Kenny	Gerry	Lenny
Betty	Gerry	Kenny	Freddy		Lenny
Conny	Lenny		Gerry	Freddy	Kenny
Dolly		Lenny		Gerry	Kenny
		Kenny	Gerry	Lenny	Freddy

	1	2	3	4	5
Freddy	Conny	Betty	Anny		
Gerry	Dolly	Anny	Betty		Conny
		Anny	Dolly	Betty	Conny
Kenny		Conny	Anny	Dolly	Betty
Lenny		Anny	Betty	Conny	Dolly

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Anny	Freddy		Kenny	Gerry	Lenny
Betty	Gerry	Kenny	Freddy		Lenny
Conny	Lenny		Gerry	Freddy	Kenny
Dolly		Lenny		Gerry	Kenny
		Kenny	Gerry	Lenny	Freddy

	1	2	3	4	5
Freddy	Conny	Betty	Anny		
Gerry	Dolly	Anny	Betty		Conny
		Anny	Dolly	Betty	Conny
Kenny		Conny	Anny	Dolly	Betty
Lenny		Anny	Betty	Conny	Dolly

Marriage Problems

	1	2	3	4	5
Anny	Freddy		Kenny	Gerry	Lenny
Betty	Gerry	Kenny	Freddy		Lenny
Conny	Lenny		Gerry	Freddy	Kenny
Dolly		Lenny		Gerry	Kenny
		Kenny	Gerry	Lenny	Freddy

	1	2	3	4	5
Freddy	Conny	Betty	Anny		
Gerry	Dolly	Anny	Betty		Conny
		Anny	Dolly	Betty	Conny
Kenny		Conny	Anny	Dolly	Betty
Lenny		Anny	Betty	Conny	Dolly

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Betty	Gerry	Kenny	Freddy		Lenny
Conny	Lenny		Gerry	Freddy	Kenny
Dolly		Lenny		Gerry	Kenny
		Kenny	Gerry	Lenny	Freddy

	1	2	3	4	5
Freddy	Conny	Betty	Anny		
Gerry	Dolly	Anny	Betty		Conny
		Anny	Dolly	Betty	Conny
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Anny	Freddy		Kenny	Gerry	Lenny
Betty	Gerry	Kenny	Freddy		Lenny
Conny	Lenny			Freddy	Kenny
Dolly		Lenny		Gerry	Kenny
		Kenny	Gerry	Lenny	Freddy

	1	2	3	4	5
Freddy	Conny	Betty	Anny		
Gerry	Dolly	Anny	Betty		
		Anny	Dolly	Betty	Conny
Kenny		Conny	Anny	Dolly	Betty
Lenny		Anny	Betty	Conny	Dolly

Marriage Problems

**College admissions and the stability of marriage,
American Mathematical Monthly 69, 1962**



David Gale



Lloyd S. Shapley

Marriage Problems: Gale-Shapley

	1	2	3	4	5
Anny	Freddy	Harry	Kenny	Gerry	Lenny
Betty	Gerry	Kenny	Freddy	Harry	Lenny
Conny	Lenny	Harry	Gerry	Freddy	Kenny
Dolly	Harry	Lenny	Freddy	Gerry	Kenny
Emmy	Harry	Kenny	Gerry	Lenny	Freddy

	1	2	3	4	5
Freddy	Conny	Betty	Anny	Emmy	Dolly
Gerry	Dolly	Anny	Betty	Emmy	Conny
Harry	Emmy	Anny	Dolly	Betty	Conny
Kenny	Emmy	Conny	Anny	Dolly	Betty
Lenny	Emmy	Anny	Betty	Conny	Dolly

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	1	2	3	4	5
Anny	Freddy ¹	Harry	Kenny	Gerry	Lenny
Betty	Gerry	Kenny	Freddy	Harry	Lenny
Conny	Lenny	Harry	Gerry	Freddy	Kenny
Dolly	Harry	Lenny	Freddy	Gerry	Kenny
Emmy	Harry	Kenny	Gerry	Lenny	Freddy

	1	2	3	4	5
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Kenny	Emmy	Conny	Anny	Dolly	Betty
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Conny	Lenny	Harry	Gerry	Freddy	Kenny
Dolly	Harry	Lenny	Freddy	Gerry	Kenny
Emmy	Harry	Kenny	Gerry	Lenny	Freddy

	1	2	3	4	5
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Harry	Emmy	Anny	Dolly	Betty	Conny
Kenny	Emmy	Conny	Anny	Dolly	Betty
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	1	2	3	4	5
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Betty	Gerry ²	Kenny	Freddy	Harry	Lenny
Conny	Lenny ³	Harry	Gerry	Freddy	Kenny
Dolly	Harry	Lenny	Freddy	Gerry	Kenny
Emmy	Harry	Kenny	Gerry	Lenny	Freddy

	1	2	3	4	5
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Betty	Gerry ²	Kenny	Freddy	Harry	Lenny
Conny	Lenny ³	Harry	Gerry	Freddy	Kenny
Dolly	Harry ⁴	Lenny	Freddy	Gerry	Kenny
Emmy	Harry	Kenny	Gerry	Lenny	Freddy

	1	2	3	4	5
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Gerry	Dolly	Anny	Betty ²	Emmy	Conny
Harry	Emmy	Anny	Dolly ⁴	Betty	Conny
Kenny	Emmy	Conny	Anny	Dolly	Betty
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Conny	Lenny ³	Harry	Gerry	Freddy	Kenny
Dolly	Harry ⁴	Lenny	Freddy	Gerry	Kenny
Emmy	Harry ⁵	Kenny	Gerry	Lenny	Freddy

	1	2	3	4	5
Freddy	Conny	Betty	Anny ¹	Emmy	Dolly
Gerry	Dolly	Anny	Betty ²	Emmy	Conny
Harry	Emmy ⁵	Anny	Dolly ⁴	Betty	Conny
Kenny	Emmy	Conny	Anny	Dolly	Betty
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Conny	Lenny ³	Harry	Gerry	Freddy	Kenny
Dolly	Harry ⁴	Lenny ⁶	Freddy	Gerry	Kenny
Emmy	Harry ⁵	Kenny	Gerry	Lenny	Freddy

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Gerry	Dolly	Anny	Betty ²	Emmy	Conny
Harry	Emmy ⁵	Anny	Dolly ⁴	Betty	Conny
Kenny	Emmy	Conny	Anny	Dolly	Betty
Lenny	Emmy	Anny	Betty	Conny ³	Dolly ⁶

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Conny	Lenny ³	Harry	Gerry	Freddy	Kenny
Dolly	Harry ⁴	Lenny ⁶	Freddy ⁷	Gerry	Kenny
Emmy	Harry ⁵	Kenny	Gerry	Lenny	Freddy

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Gerry	Dolly ⁸	Anny	Betty ²	Emmy	Conny
Harry	Emmy ⁵	Anny	Dolly ⁴	Betty	Conny
Kenny	Emmy	Conny	Anny	Dolly	Betty
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Marriage Problems: Gale-Shapley

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Betty	Gerry ²	Kenny ⁹	Freddy	Harry	Lenny
Conny	Lenny ³	Harry	Gerry	Freddy	Kenny
Dolly	Harry ⁴	Lenny ⁶	Freddy ⁷	Gerry ⁸	Kenny
Emmy	Harry ⁵	Kenny	Gerry	Lenny	Freddy

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Gerry	Dolly ⁸	Anny	Betty ²	Emmy	Conny
Harry	Emmy ⁵	Anny	Dolly ⁴	Betty	Conny
Kenny	Emmy	Conny	Anny	Dolly	Betty ⁹
Lenny	Emmy	Anny	Betty	Conny ³	Dolly ⁶

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Betty	Gerry ²	Kenny ⁹	Freddy	Harry	Lenny
Conny	Lenny ³	Harry	Gerry	Freddy	Kenny
Dolly	Harry ⁴	Lenny ⁶	Freddy ⁷	Gerry ⁸	Kenny
Emmy	Harry ⁵	Kenny	Gerry	Lenny	Freddy

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Freddy	Conny	Betty	Anny ¹	Emmy	Dolly ⁷
Gerry	Dolly ⁸	Anny	Betty ²	Emmy	Conny
Harry	Emmy ⁵	Anny	Dolly ⁴	Betty	Conny
Kenny	Emmy	Conny	Anny	Dolly	Betty ⁹
Lenny	Emmy	Anny	Betty	Conny ³	Dolly ⁶

Marriage Problems: Gale-Shapley

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Conny	Lenny	Harry	Gerry	Freddy	Kenny
Dolly	Harry	Lenny	Freddy	Gerry	Kenny
Emmy	Harry	Kenny	Gerry	Lenny	Freddy

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Gerry	Dolly	Anny	Betty	Emmy	Conny
Harry	Emmy	Anny	Dolly	Betty	Conny
Kenny	Emmy	Conny	Anny	Dolly	Betty
Lenny	Emmy	Anny	Betty	Conny	Dolly

Marriage Problems: Gale-Shapley

Gale-Shapley algoritme:

- Gives best stable matching for the “proposers”
- Also applicable when groups have different size
- Also applicable if staying single is permitted
- Also applicable for “college admissions”

Stability among Widows: Bob Aumann



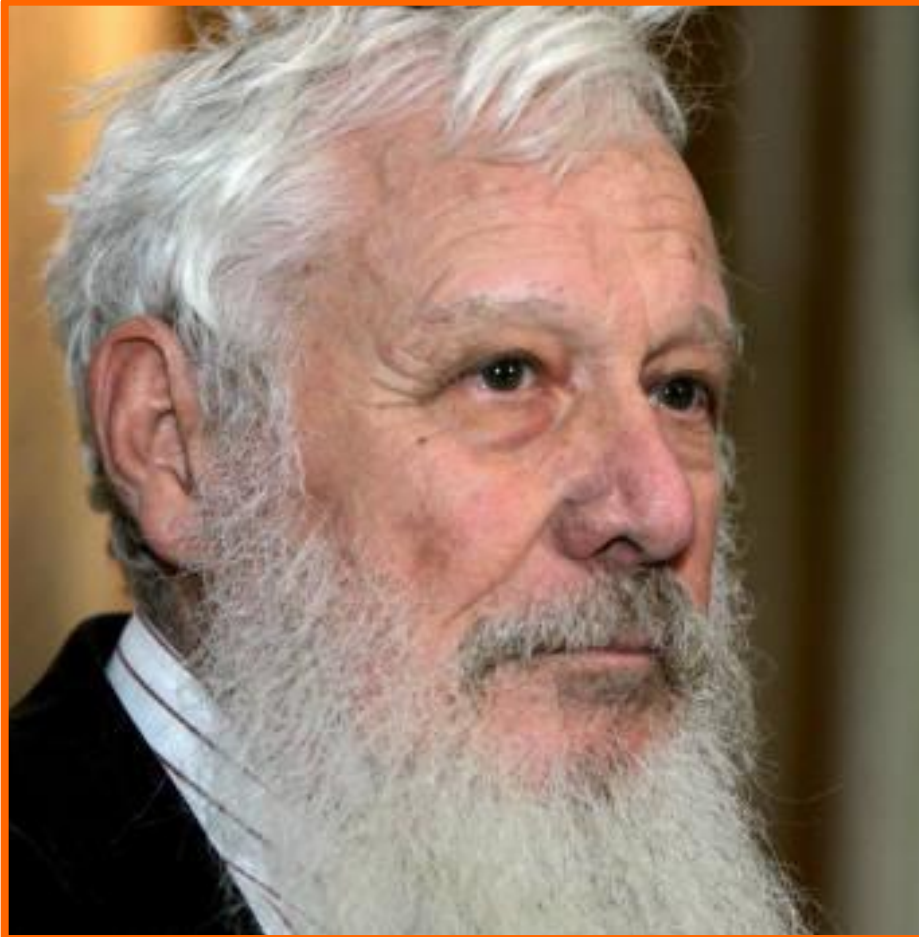
2005



Thomas Schelling

Bob Aumann

Game theoretic analysis of a bankruptcy problem from the Talmud, *Journal of Economic Theory* 36, 1985



Bob Aumann



Michael Maschler

A problem of rights arbitration from the Talmud, *Mathematical Social Sciences* 2, 1982



Barry O'Neill

A Bankruptcy Problem from the Talmud

Kethuboth, Fol. 93a, Babylonian Talmud, Epstein, ed, 1935

“If a man who was married to three wives died and the kethubah of one was 100 zuz, of the other 200 zuz, and of the third 300 zuz, and the estate was worth only 100 zuz, then the sum is divided equally.

If the estate was worth 200 zuz then the claimant of the 100 zuz receives 50 zuz and the claimants respectively of the 200 and the 300 zuz receive each 75 zuz.

If the estate was worth 300 zuz then the claimant of the 100 zuz receives 50 zuz and the claimant of the 200 zuz receives 100 zuz while the claimant of the 300 zuz receives 150 zuz.

Similarly if three persons contributed to a joint fund and they had made a loss or a profit then they share in the same manner.”

A Bankruptcy Problem from the Talmud

Estate

Widow

	100	200	300
100	33.33		
200	33.33		
300	33.33		

A Bankruptcy Problem from the Talmud

Kethuboth, Fol. 93a, Babylonian Talmud, Epstein, ed, 1935

"If a man who was married to three wives died and the kethubah of one was 100 zuz, of the other 200 zuz, and of the third 300 zuz, and the estate was worth only 100 zuz, then the sum is divided equally.

If the estate was worth 200 zuz then the claimant of the 100 zuz receives 50 zuz and the claimants respectively of the 200 and the 300 zuz receive each 75 zuz.

If the estate was worth 300 zuz then the claimant of the 100 zuz receives 50 zuz and the claimant of the 200 zuz receives 100 zuz while the claimant of the 300 zuz receives 150 zuz.

Similarly if three persons contributed to a joint fund and they had made a loss or a profit then they share in the same manner."

A Bankruptcy Problem from the Talmud

Estate

Widow

	100	200	300
100	33.33	50	
200	33.33	75	
300	33.33	75	

A Bankruptcy Problem from the Talmud

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Similarly if three persons contributed to a joint fund and they had made a loss or a profit then they share in the same manner.”

A Bankruptcy Problem from the Talmud

Estate

Widow

	100	200	300
100	33.33	50	50
200	33.33	75	100
300	33.33	75	150

A Bankruptcy Problem from the Talmud

		Estate		
Widow		100	200	300
	100	33.33	50	50
	200	33.33	75	100
	300	33.33	75	150
		Equal	???	Proportional

A Bankruptcy Problem from the Talmud

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A Bankruptcy Problem from the Talmud

Estate

Widow

	100	200	300
100	33.33	50	50
200	33.33	75	100
300	33.33	75	150

Equal ??? Proportional

"Other amounts shared **in the same way.**" ?????

A Bankruptcy Problem from the Talmud

Estate

Widow		100	200	300
	100	33.33	50	50
	200	33.33	75	100
	300	33.33	75	150

Equal ??? Proportional

"Other amounts shared **in the same way**." ?????

How to share 400?

A Bankruptcy Problem from the Talmud

Estate

	100	200	300
100	33.33	50	50
200	33.33	75	100
300	33.33	75	150

Widow

Equal ??? Proportional

"Other amounts shared **in the same way**." ?????

How to share 400?

What if a fourth widow claims 400?

The Answer

Baba Metzia 2a, Fol. 1, Babylonian Talmud, Epstein, ed, 1935

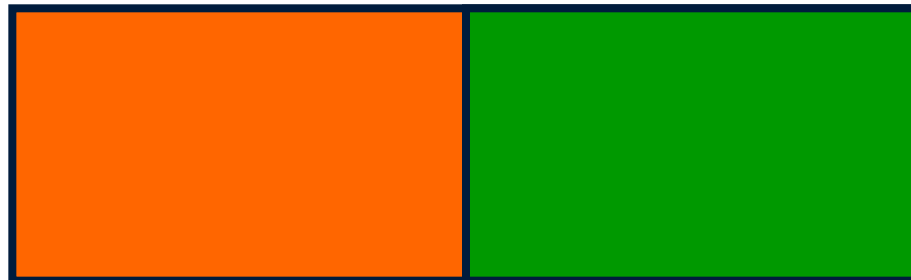
“Two hold a garment;
one claims it all, the other claims half.
Then one gets $\frac{3}{4}$, while the other gets $\frac{1}{4}$.”



The Answer

Baba Metzia 2a, Fol. 1, Babylonian Talmud, Epstein, ed, 1935

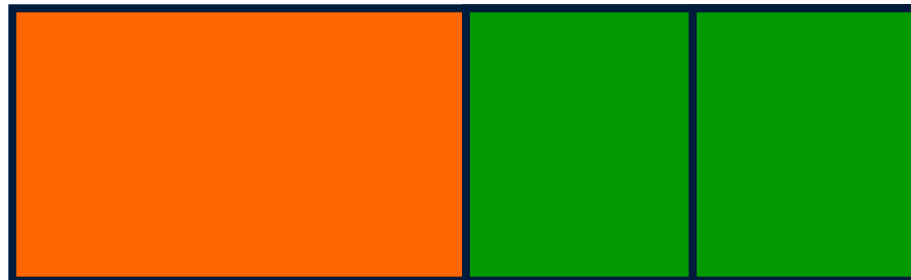
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The Answer

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“Two hold a garment;
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Consistency

		Estate		
Widow		100	200	300
	100	33.33	50	50
	200	33.33	75	100
	300	33.33	75	150

A closer look at these numbers

Consistency

		Estate		
Widow		100	200	300
	100	33.33	50	50
	200	33.33	75	100
	300	33.33	75	150

Consistency

		Estate		
Widow		100	200	300
	100	33.33	50	50
	200	33.33	75	100
	300	33.33	75	150

Consistency

		Estate		
Widow		100	200	300
	100	33.33	50	50
	200	33.33	75	100
	300	33.33	75	150

Consistency

		Estate		
Widow		100	200	300
	100	33.33	50	50
	200	33.33	75	100
	300	33.33	75	150

Consistency

		Estate		
Widow		100	200	300
	100	33.33	50	50
	200	33.33	75	100
	300	33.33	75	150

The questions remain:

How to share 400?

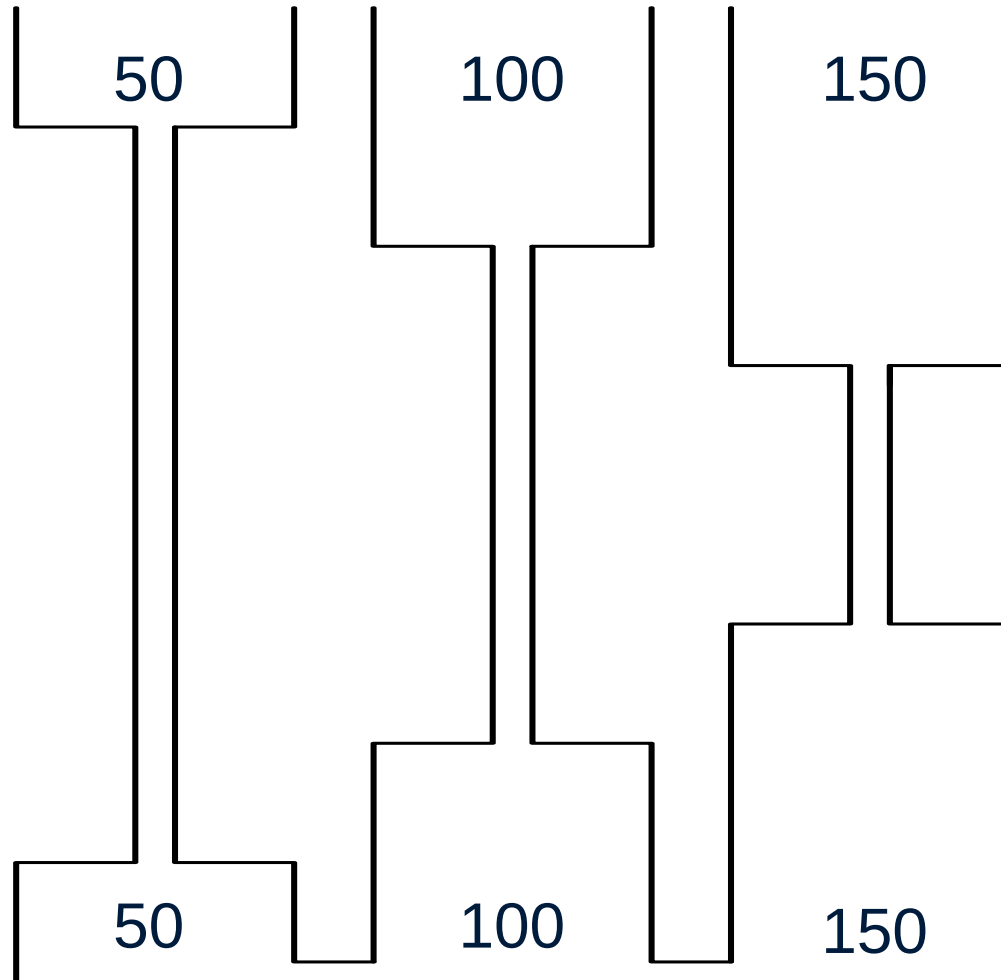
What if a fourth widow claims 400?

'Hydraulic' rationing, Mathematical Social Sciences 40, 2000

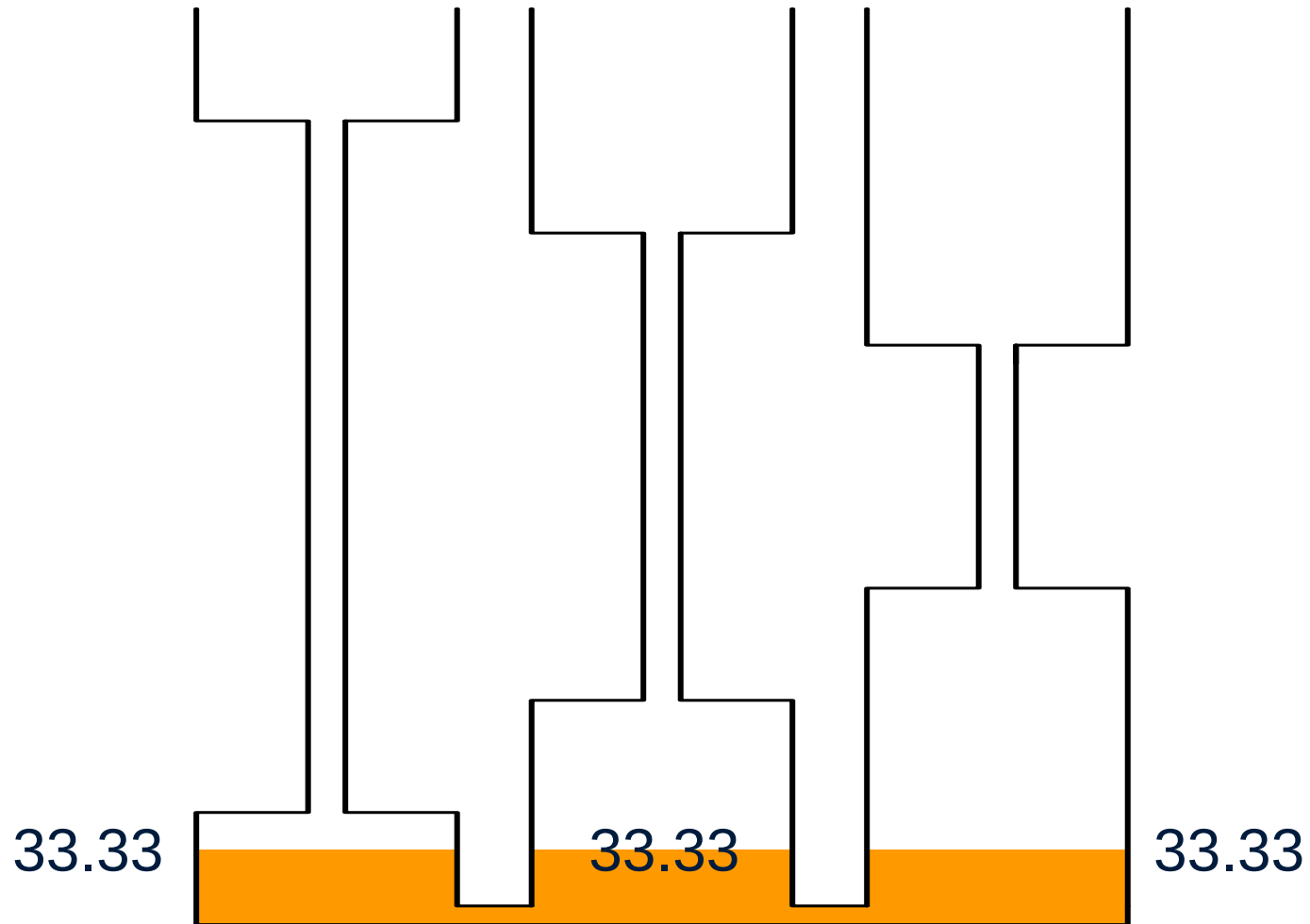


Marek M. Kaminski

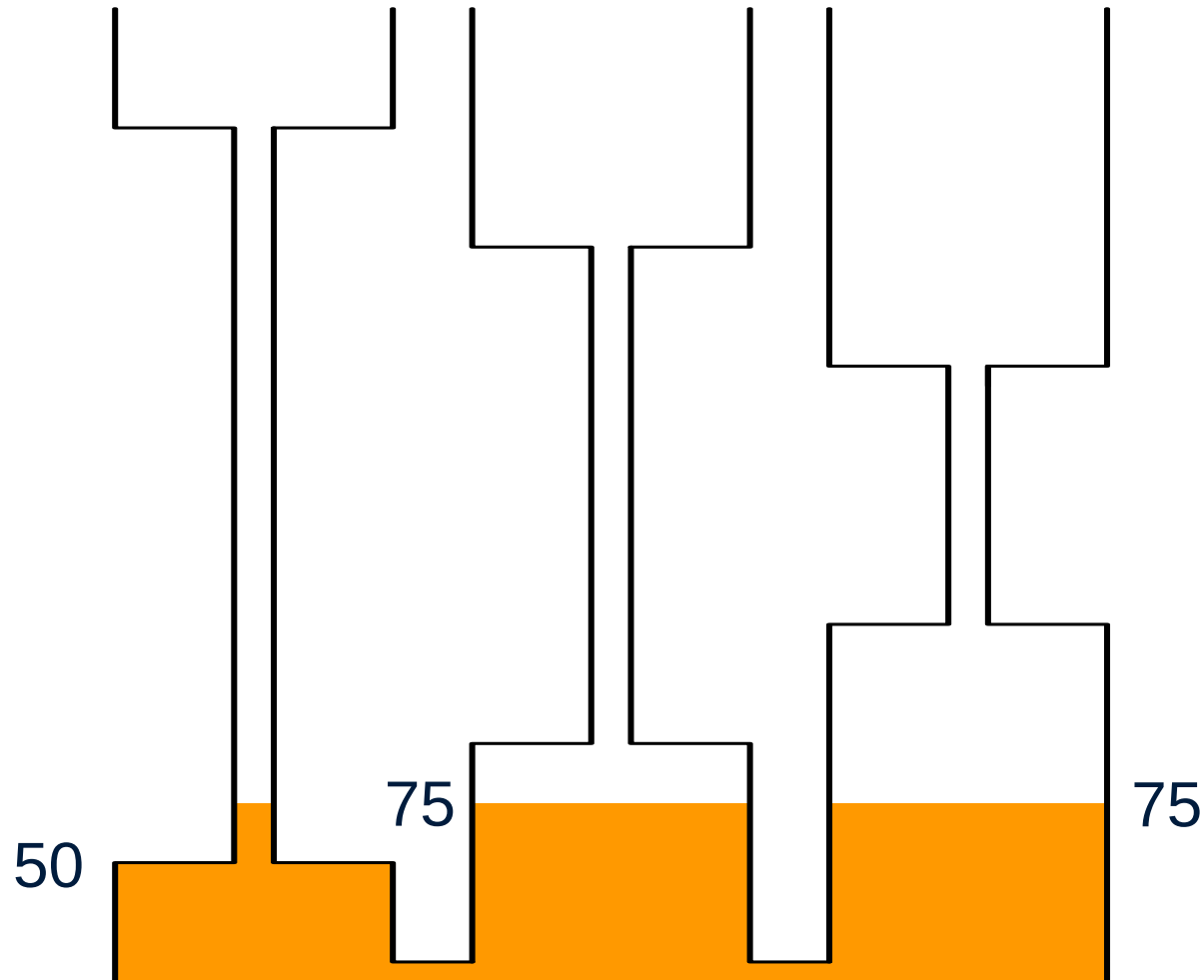
Communicating Vessels



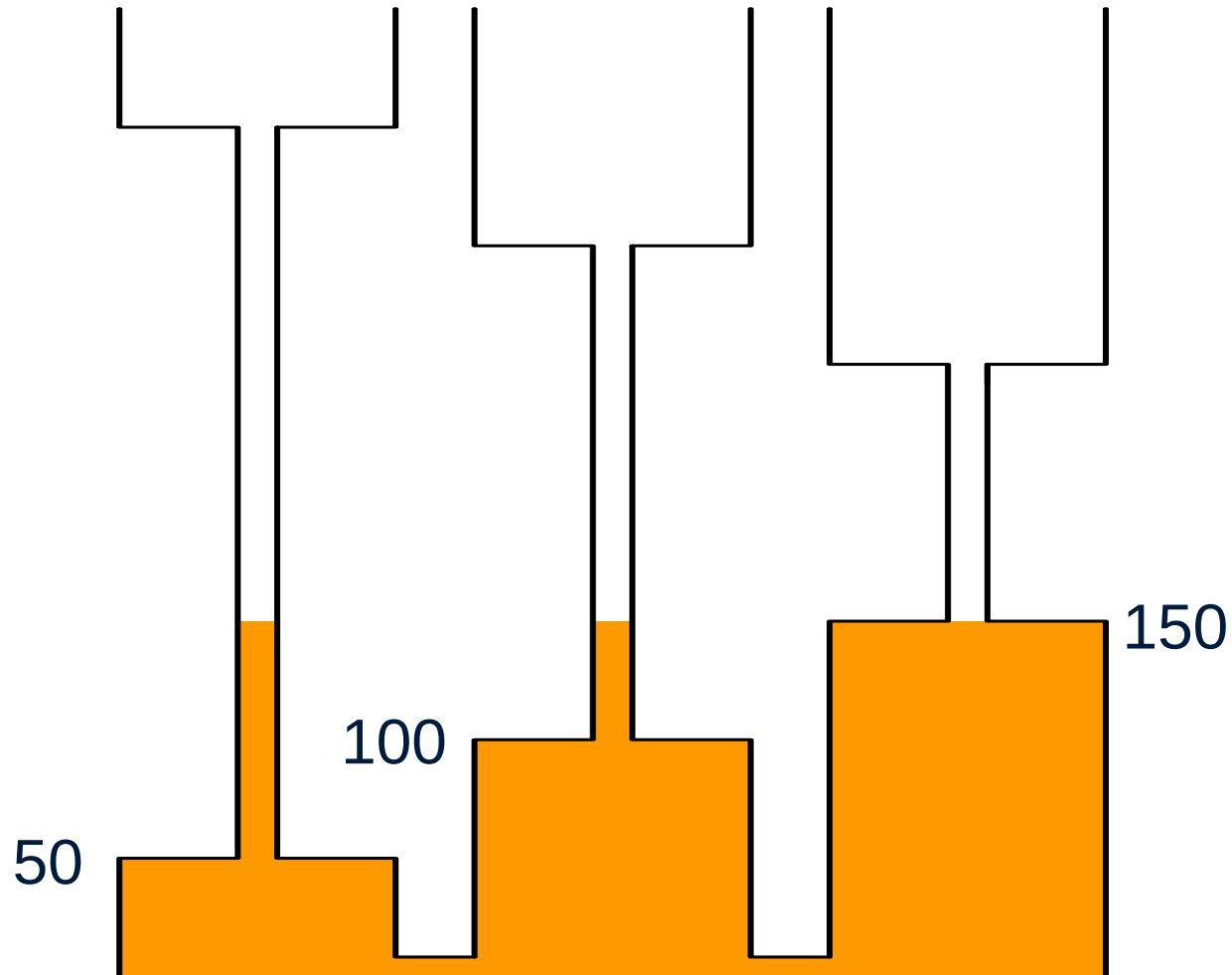
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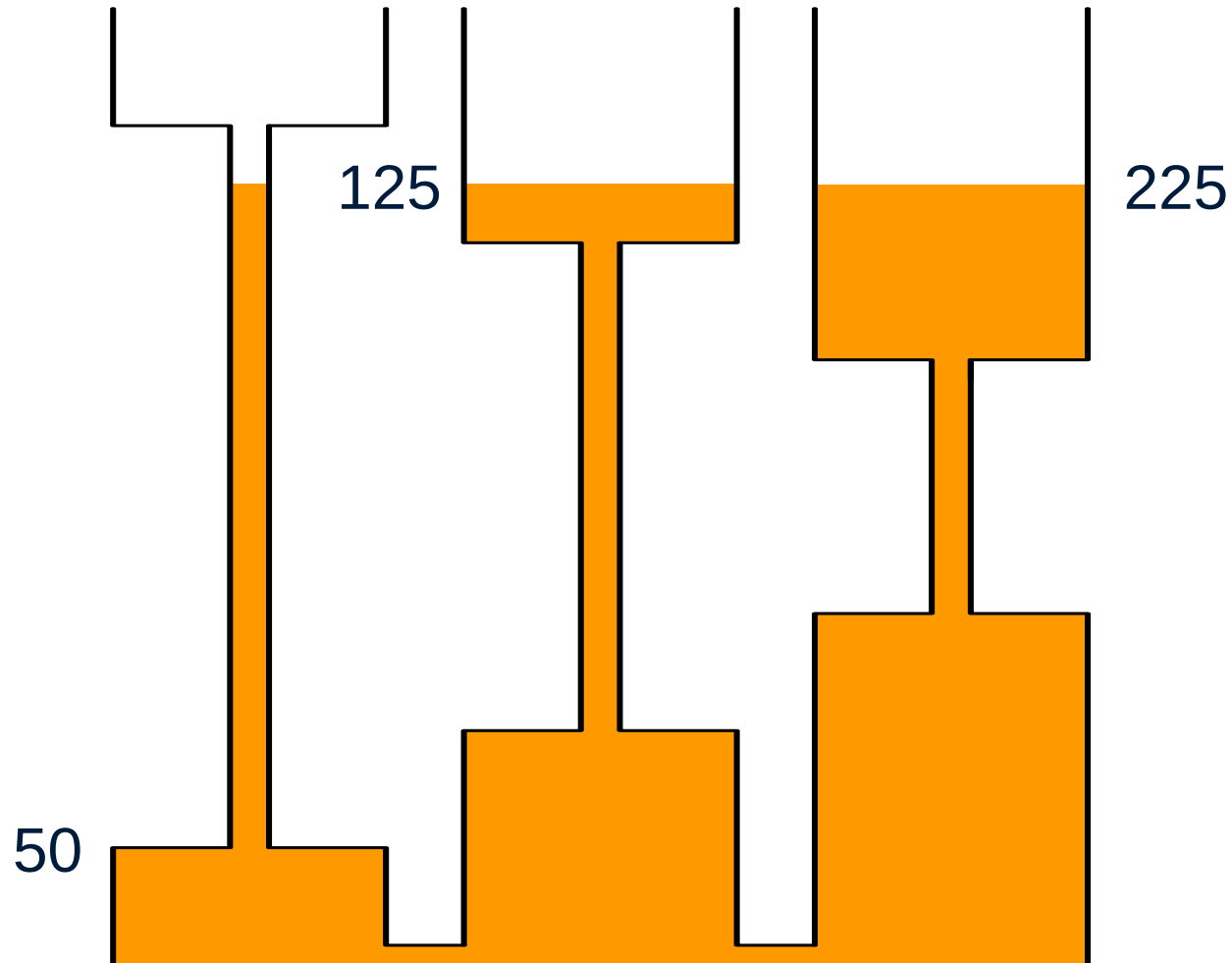
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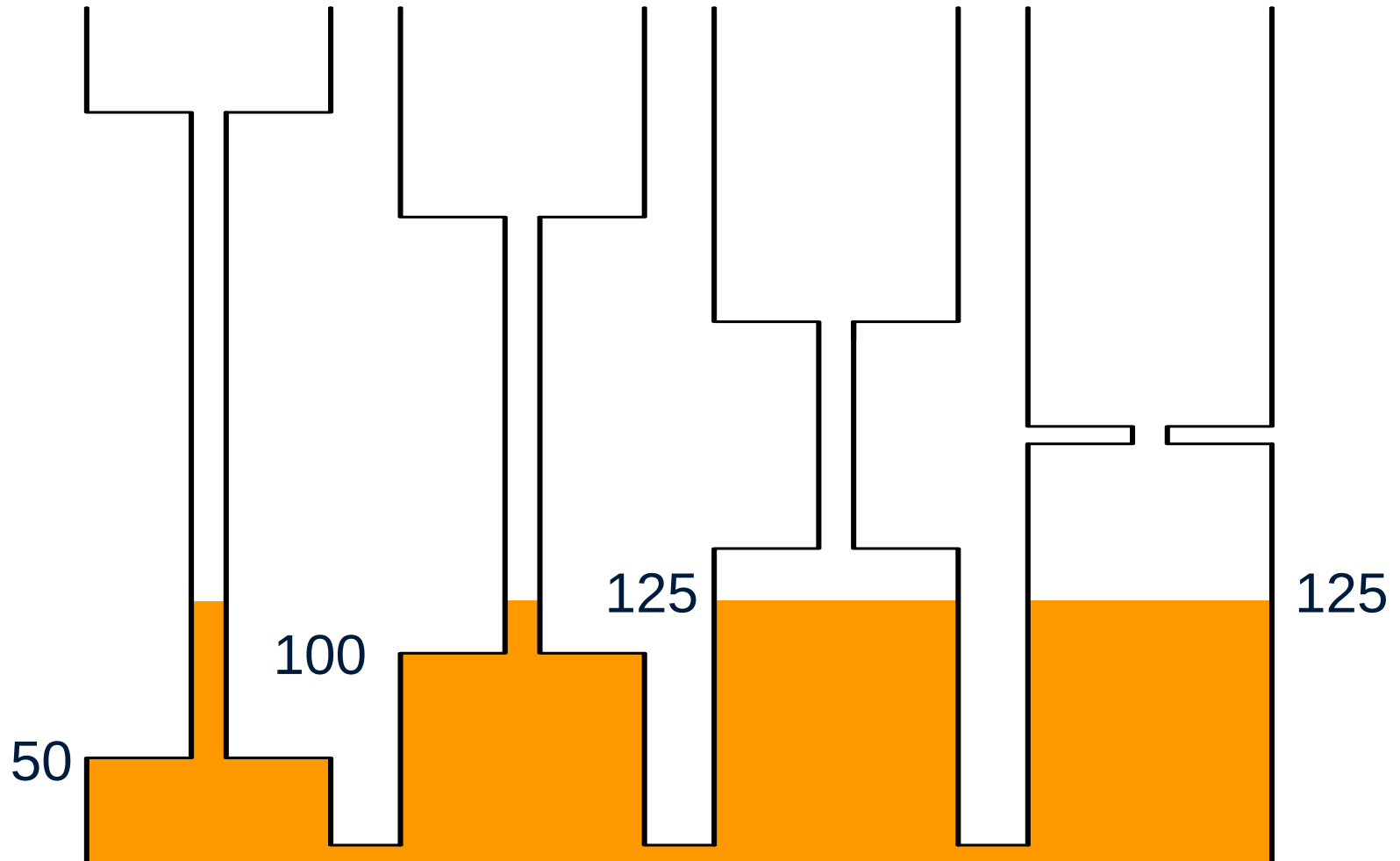
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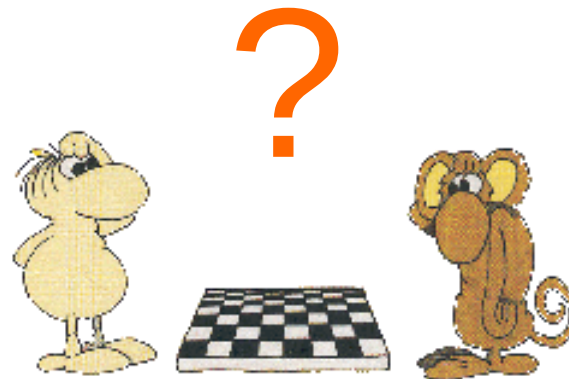
Communicating Vessels: 400



Communicating Vessels: 400 for 4



Many Thanks for Your Attention!



Just for Fun

