From the Talmud into Present-Day Politics
- A Bankruptcy Problem Approach to EU Fisheries Management -

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Abstract

Bankruptcy Problems provide a broad opportunity of mathematical analysis as the solution methods to these problems are plentiful, especially after rewriting the problem as a cooperative game. The latter leads to several unique one-point solutions such as the Shapley value and nucleolus, which coincides with the n-person Contested Garment solution developed to solve the Talmudic Widow Problem, a specific Bankruptcy Problem. Moreover, the theoretical matter of Bankruptcy Problems and their various allocation rules can prove its applicability in many real-life situations. Especially in an era of collapsing fish stocks the European Union fisheries management is one of these fields. Formulating the division of fishing rights amongst fisheries as a Bankruptcy Problem with the pre-set Total Allowable Catch as the estate, two specific scenarios are brought forward. Firstly, an investigation of which allocation model is best capable of bringing the fish stocks back to a sustainable situation – considering the selectivity of the coastal vs. trawler fleet as the models' core parameter – prioritizes the Constrained Equal Awards rule above the currently adopted proportional rule. Furthermore, in situations where new fisheries plan on entering an existing system (i.e., the New Member Problem) a population monotonic division scheme (PMAS) is amongst one of the suggested solutions that maintain the system's stability. Finally, justice or fairness is of paramount importance in the context of Bankruptcy Problems. Only when based on an extensive analysis of relevant properties a choice of allocation rules should be made, something that could be argued is not addressed properly in the current management system of the European Commission Fisheries.

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Chapter 1

Introduction

"If a man was married to three women and died, and if the amount of the Ketubah was 100 zuz for the first, 200 zuz for the second, and 300 zuz for the third, and if his estate only contains 100 zuz, they divide it equally. If the estate contains 200 zuz, the one with the 100-zuz Ketubah gets 50, and the other two get 75 each. If the estate contains 300 zuz, the one with the 100-zuz Ketubah gets 50, the one with the 200-zuz Ketubah gets 100, and the one with the 300-zuz Ketubah gets 150. And if three people [similarly] contributed to a fund, and it lost or gained, this is how they divide things."

(Kethuboth chapter 1, Mishna 3, Babylonian Talmud)

תלמוד בבלי, מסכת כתובות דף צג/א

מתני' מי שהיה נשוי שלש נשים ומת,
כתובתה של זו מנה ושל זו מאתים
ושל זו שלש מאות ואין שם אלא מנה, חולקין בשוה.
היו שם מאתים, של מנה נוטלת חמשים,
של מאתים ושל שלש מאות שלשה שלשה של זהב.
היו שם שלש מאות, של מנה נוטלת חמשים ושל
מאתים מנה ושל שלש מאות ששה של זהב.
וכן ג' שהטילו לכיס פיחתו או הותירו כך הן חולקין.

1.1 A Talmudic Widow Problem – Mathematics in Jewish Scripts

The excerpt from the Babylonian Talmud on the beginning of this page[1] summarizes a way of allocating an estate (viz. 100, 200 or 300 units or 'zuz') to three widows claiming respectively 100, 200 and 300 units from their deceased husband. However, it is not easy to trace back the algorithm behind this allocation proposed by Rabbi Nathan, as will be made clear in the following pages. This specific division of the bequest – which will be called the 'Widow Problem' from now on – therefore provides an interesting basis for mathematical analysis.

Looking at the source of the problem in order to get to know the context of the passage, the Talmud is a combination of two major works in the history of Jewish law. The oldest work, *Mishna* (translated: 'learning, revision'), is a collection of Jewish juridical guidelines or laws that were not yet officially written down at the time, which was done by rabbi Jehuda ha-Nasi around 135-217 A.D. Commentaries on and additions to this work formed the *Gemara* (translated: 'addition'). There are again two main versions of the Talmud, i.e. the Babylonian and Jerusalem or Palestine Talmud that differ in the Gemara-part. All in all, it can be assumed that the advice given by rabbi Nathan concerning the allocation of estates amongst widows has been seriously considered as a guideline, however the commentaries do show different interpretations of the text.

[1] All translations are taken from: Rosenberg, J. *Some Examples of Mathematical Analysis Applied to Talmud Study*. Retrieved April 12, 2008, from http://www.math.umd.edu/~jmr/MathTalmud.html#note4.

To come back at the content of the Mishna, looking at only the first and third column of the table below, it doesn't require much investigation to notice that the first allocation (100 zuz) follows the rule of equal division, whereas the division in the case of 300 zuz is proportional. However, the second column of 200 zuz seems at first sight inconsistent. Still, the concluding sentence of the Mishna above indicates that there exists some kind of rationale that is able to explain all three allocations.

		ESTATE		
		100	200	300
CLAIM	100	33 1/3	50	50
	200	33 1/3	75	100
	300	33 1/3	75	150

Table 1.1. The Widow Problem in Tabular Form

During the centuries, many have tried to find a solution (even mistakes in translation were suggested) but it wasn't until 1985 that a first publication was made in which a consistent rule covering the three instances was brought forward by Aumann & Maschler. A few years previously to this article, O'Neill (1982) was the first to describe another problem from the Talmud that was closely related to the Widow Problem (which was therefore indeed pointed at in a note) and he provided several ways of division that can be found back in the Jewish scripts. The rules they proposed and additionally various allocation rules explaining (parts of) the problem will be presented in the following. However, first it is important to generalize this Talmudic Widow Problem to a more general case, namely the class of Bankruptcy Problems.

1.2 The Bankruptcy Problem and its Application Areas

Whereas the Widow Problem provides several examples of how to allocate three estates of different amounts to three claimants, there are obviously infinitely many different situations to think of that also follow a similar structure. Regardless of the methods used to derive an allocation, in each Bankruptcy Problem there are n claimants with known claims and an estate E that is smaller than the total sum of claims. A formal definition of a Bankruptcy Problem will be provided in the next chapter.

An interesting aspect about the Bankruptcy Problem is that it has many different fields of application. In many situations in which (common) property needs to be divided, a Bankruptcy Problem can be discovered. Therefore this type of problem is more generally called 'simple claims problem' by O'Neill (1982) or rationing problem by Kaminski (2000), of which the bankruptcy or bequest problem is just one interpretation. Other examples of such problems are situations of taxation, cost allocation or surplus sharing problems and aircraft landing problems.

Another field with a less obvious relation to the applications just mentioned is the fisheries management of the European Union, which will be analyzed in this paper at a later stage. In fact, there are multiple levels in fisheries management where a bankruptcy formulation can be constructed, e.g., in taking specific (types of) fisheries or member states as claimants of the amount of fish that can be caught. Furthermore, in the formulation of claims either the ecological state of the fish stocks or the economical conditions of fisheries or nation states could be taken into consideration. These and other aspects make this situation a complex one that moreover clearly shows the interdisciplinary character of the problem.

Apart from revealing situations from real-life that can be interpreted as Bankruptcy Problems, also the division or allocation algorithms available to solve these problems are evidently of interest. Along with any sharing problem the question will consequently arise of what division is a fair one. This aspect is an important determinant on which the choice amongst allocation algorithms will be based, giving the theoretical models a more 'soft' basis of discussion.

1.3 Structure of the Paper

The aim of this thesis is twofold: in the first part of the paper the concept of Bankruptcy Problems will be discussed alongside various allocation rules, taking the Talmudic Widow Problem as an example. In the latter part the situation of the EU fisheries management will be analyzed and suggestions for improvement will be made that resulted from implementations of the Bankruptcy Problem.

In more detail, chapter 2 will contain several allocation rules that are applicable to the Widow Problem. In chapter 3 the Bankruptcy Problem will be rewritten into a so-called coalitional or cooperative game on which basis another pair of solution methods can be added to the list. The final chapter of the first part will cover the question of what makes a fair allocation, based on an analysis of the properties of the various rules. Moving to part II of the paper, chapter 5 will provide an introduction to the situation in global and European seas and will be followed by an introduction in the management structure of the European Commission Fisheries. In chapter 6 some applications of the Bankruptcy Problem are made. In specific, a dynamic bankruptcy model is implemented to arrive at an ecologically sustainable situation and the Bankruptcy Problem is re-formulated as a New Entrant Problem, providing solutions to a situation where new fishing parties join the established group of fisheries. Finally, results and findings will be summarized in the concluding chapter of this paper.

Part I
The Bankruptcy Problem and Allocation Rules

Now that the concept of a Bankruptcy Problem has been introduced, in this part of the paper an overview of allocation rules for these kinds of problems is provided. This inventory of rules is split-up into the first two chapters where the former focuses on some common allocation rules and the Talmudic Contested Garment principle that provides a solution to the Widow Problem as depicted by the Talmudic table. The latter chapter presents methods to derive unique one-point solutions to Bankruptcy Problems that are interpreted as cooperative games consisting of coalitions rather than individual claimants. This first part will be concluded with a chapter in which properties of the allocation algorithms will be analyzed, in order to explain people's preferences for certain methods above others.

Chapter 2

An Inventory of Allocation Rules

Coming back at the Talmudic table, the second instance considering an inheritance of 200 may seem incompatible with any consistent rule and a clear explanation isn't found in any Mishna, as is underlined in the Tosafot to Kethubot 93: 'The matter is not clear.' (Kitrossky, 2001). However, staying close to the place of origin of the Widow Problem, the Talmud comprises more interesting clues that lead to a solution. However, apart from searching for 'The One Rule' able to explain all allocations, it is already interesting to compare various allocation principles that only partly give an explanation of the Talmudic division scheme.

In this chapter, next to a short description of the rule of equal division and the proportional method, the 2-person Contested Garment principle from the Talmud that was formally described by Aumann and Maschler (1985) will be presented. The following section will then move further to the *n*-person solution that is consistent with the Contest Garment principle. Next, the Constrained Equal Awards and Losses methods are described, making the link back to the *n*-person solution. After having shown a recent publication in the research about Bankruptcy Problems, i.e. the intuitive hydraulic representation that was developed by Kaminski in 2000, the chapter will be ended by standing still at the notion of justice in relation to allocation algorithms.

However, first and foremost the formal definition of a Bankruptcy Problem is needed, which can be presented as follows:

> A Bankruptcy Problem is denoted $(E \mid d_1, d_2, ..., d_n)$ where the claims of party i are denoted d_i and the estate is represented by E. Furthermore, $0 \le E \le d_1 + d_2 + ... + d_n$ or more accurately $0 \le E < d_1 + d_2 + ... + d_n$. Allocations are given as the vector $x = \{x_1, x_2, ..., x_n\} \in \Re^n$ where x_i provides the allocation to party i and $\sum_i x_i = E$ (i.e., the allocation is *efficient*).

2.1 Equal Division and the Proportional Method

Two possible allocation rules that are able to explain the first and third column of the Talmudic table are fairly obvious: the estate of 100 is divided equally amongst the claimants no matter the height of their individual claims and the estate of 300 is allocated proportional to the claims, i.e. 50 (100), 100 (200) and 150 (300). It follows self-evidently how these methods can be applied to Bankruptcy Problems of any number of claimants, claim amount and value of the estate.

It is not surprising that the proportional method is also described in one of the Jewish scripts. Levi Kitrossky (2001) mentioned that this procedure was presented in Kethubot 93 ('Tosafot') by Rabbi Omer.

2.2 A Solution from the Talmud

In order to find the mechanism behind the Widow Problem, Aumann and Maschler (1985) selected a method explained in another Mishna. Subsequently they combined this 2-person principle with another cue from the Talmud in order to arrive at an *n*-person solution that not only explains the Talmudic table, yet is an interesting solution to any Bankruptcy Problem.

2.2.1 The Contested Garment Principle

"If two people are holding on to a garment, one saying "I found it" and the other saying "I found it", or one saying "it's all mine" and the other saying "it's all mine", the one should swear he owns at least a half, and the other should swear he owns at least a half, and they should divide it. If one says "it's all mine" and the other says "it's half mine", the one who says "it's all mine" should swear he owns at least a half, and the one who says "it's half mine" should swear he owns at least a quarter, and they should divide it in the ratio 3:1."

(Baba Metzia chapter 1, Mishna 1, Babylonian Talmud)

משנה, מסכת בבא מציעא, פרק א, משנה א

שנים אוחזין בטלית, זה אומר אני מצאתיה
וזה אומר אני מצאתיה,
זה אומר כלה שלי וזה אומר כלה שלי,
זה ישבע שאין לו בה פחות מחציה,
וזה ישבע שאין לו בה פחות מחציה, ויחלוקו.
זה אומר כלה שלי וזה אומר חציה שלי,
האומר כלה שלי, ישבע שאין לו בה פחות משלשה חלקים.
והאומר חציה שלי, ישבע שאין לו בה פחות מרביע.
זה נוטל שלשה חלקים, וזה נוטל רביע:

This second text excerpt taken from the Talmud by Aumann & Maschler (1985) gives a first clue to what a possible allocation mechanism could be. Named the 'Contested Garment (CG) principle', this situation can be depicted as in figure 2.2.1, where two people are holding a piece of cloth from one side and both claim a certain part of it. The solution suggested in the Talmud only considers the part of the garment that both parties are fighting on, i.e. that is *contested.* This part will be divided equally amongst both claimants, whereas the uncontested part(s), namely from each side up till the contested part, go to the claimant(s) on the particular side(s) of the cloth. In figure 2.2.1 this uncontested part corresponds to the right half; it can be seen from the arrow going rightward – claiming only the left half – that the right half is not of interest to the smaller claimant and can be given to its counterpart. Thus, the total amount allocated to claimant *i* is: $x_i = \frac{E-(E-d_1)_+ - (E-d_2)_+}{2} + (E-d_j)_+$, where $(...)_+ = \max((...),0)$.

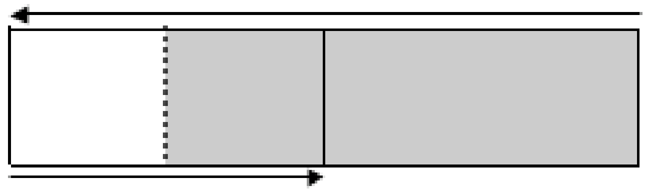

Figure 2.2.1. The Contested Garment Principle

The Contested Garment principle is interesting because, looking back at the Widow Problem, for each pair of claimants (*i* and *j*) taken out of the Talmudic table, the values allocated to them that result from applying the CG principle to the estate left to divide ($E - d_k$) correspond to those in the table (where the three claimants are denoted *i*, *j* and *k* in random order). Thus, the CG principle can be applied to Bankruptcy Problems with any number of claimants, as will be explained in more detail in the following section.

2.2.2 An *n*-Person Solution

"...This is when the third widow authorizes the second to deal with the first. Then she [the second] says to her [the first]: you are only claiming 100 [and the two of us are claiming the same 100], so take 50 and go."

(Jerusalem Talmud)

תלמוד ירושלמי, מסכת כתובות, פרק י', הלכה ד'

שמואל אמר במרשות זו את זו
כשהרשת השלישית את השנייה לדון עם הראשונה.
אמרה לה, לא מנה אית לך, סב חמשין ואיזל לך.

In the situation outlined above a somewhat hierarchical system can be detected. At first, the two parties claiming most form a so-called coalition with a total claim overshadowing the party left; a part of the estate is given to this claimant and in the next step the remaining parties stand against each other to determine the division of the estate left. Generalizing this situation, the most important aspect is that at each stage of the allocation process, the lowest-ranked claimant contests with the remaining parties that act as one coalition of claimants. Aumann and Maschler (1985) combined this idea with the Contested Garment principle, interpreting the problem as a multistage 'event', eventually leading to an allocation that is conform to the Talmudic table.

However, some refinements should be made first to this simple procedure in order to cover the following situations. It would not be fair to follow the standard CG procedure if:

1) A claimant would be given so much that at least one of the remaining claimants (thus, claimants with larger claims) will receive less. Instead it is proposed to share the remaining estate equally.
2) A lower claimant loses more than a party claiming more (where loss is defined as the claim minus the allocated part of the estate). Instead it is proposed to share the loss equally.

In short, assuming a Bankruptcy Problem as formulated in the introduction of this paper, $(E \mid d_1, d_2, ..., d_n)$, at first the claimants are ordered by increasing amounts of claims:

$d_1 \le d_2 \le ... \le d_n$. Furthermore the solution vector $x = (x_1, x_2, ..., x_n)$ should correspond to $x_1 \le x_2 \le ... \le x_n$ and to $d_1 - x_i \le d_2 - x_2 \le ... \le d_n - x_n$ (i.e. the solution is *order preserving*).
Now, in each step the Consistent Garment principle is applied by comparing the claim of party *i* (i.e., the smallest claimant in that step) with those of the coalition of parties {*i+1*, ..., *n*} (all remaining claimants), where the remaining amount to be divided is E_i.:

If $\frac{d_i}{2} \le \frac{E_i}{n-i+1} \le \frac{d_i + d_{i+1} + ... + d_n}{n-i+1} - \frac{d_i}{2}$, the standard CG rule says that party *i* should receive $\frac{d_i}{2}$ and the remaining $(E_i - \frac{d_i}{2})$ will be allocated to the *n* – *i* claimants left.

1) If $\frac{d_i}{2} \ge \frac{E_i}{n-i+1}$ as described in the former of the exceptional situations, each of the claimants left at that stage receive an equal amount of $\frac{E_i}{n-i+1}$.

2) If $\frac{E_i}{n-i+1} \ge \frac{d_i + d_{i+1} + ... + d_n}{n-i+1} - \frac{d_i}{2}$, as described in the latter of the exceptional situations, each of the claimants left at that stage suffers an equal amount of loss, resulting in an allocation of $d_j - \frac{d_i + d_{i+1} + ... + d_n - E_i}{n-i+1}$, where d_j is the claim of the individual claimant considered.

This algorithm completes the search for a consistent allocation method able to solve the Widow Problem. Using a coalitional approach it can be shown that this method is consistent with any number of claimants and any value for the estate; it is in the same manner applicable to all three allocations in the Widow Problem and it can be used to further extend this table to include either more estate values and/or claimants. The question whether this makes a *fair* division is still an open issue though.

2.3 Constrained Equal Awards and Constrained Equal Losses

Two further allocation principles that are consistent with any two-party situation from the Talmudic table are the closely related Constrained Equal Awards (CEA) and Constrained Equal Losses (CEL) rules.

Following the Constrained Equal Awards rule, all parties are allocated an equal amount of the estate, unless the claim of one of them has been reached. In that case, the remainder of the estate is divided in a similar way to the other parties that still have an unfulfilled claim. In other words, each is given $\frac{E}{n}$ until $d_i \le \frac{E}{n}$, in which case *E* – d_i is divided amongst remaining claimants *i*+1 until *n*.

Opposite to this division rule, in Constrained Equal Losses the differences between the claims and amounts that are allocated are equal to each of the claimants.

Two interesting things should be noted at this point. First, the CG-consistent solution to a Bankruptcy Problem where $E \le \frac{1}{2}\sum_i^n d_i$ is the same as the solution of $(E \,|\, \frac{1}{2}d_1, \frac{1}{2}d_2, ..., \frac{1}{2}d_n)$ using CEA. A similar idea of using half of each claim while adopting CEL is applicable to situations in which the estate exceeds a half of the total claim. Now, the amount to divide amounts to the estate minus this total claim, where a half of each individual claim is added to the allocations resulting from applying CEL to $(E'|\ \frac{1}{2}d_1, \frac{1}{2}d_2, ..., \frac{1}{2}d_n)$, where $E' = E - \frac{1}{2}\sum_i^n d_i$.

Moreno-Ternero and Villar (2006) even go as far as defining a 'TAL-family of rules' where by choosing parameter $\theta \in [0,1]$, allocation rule R^θ makes sure that in case $E \leq \theta \sum_i^n d_i$ no claimant will receive more than $\theta * d_i$ and similarly, no claimant will receive less than this amount if the estates exceeds fraction θ of the sum of claims. Thus, for θ = 0 and θ = 1 the allocation rules of respectively Constrained Equal Losses and Constraint Equal Awards will be adopted, whereas the *n*-person solution described in section 2.2.2 can be obtained for $\theta = \frac{1}{2}$, and along with that an infinite number of rules exists for different values of θ. The following figure (Moreno-Ternero & Villar, 2006, p.6) shows the allocations in a situation of two claimants for various θ (T = Talmudic *n*-person solution).

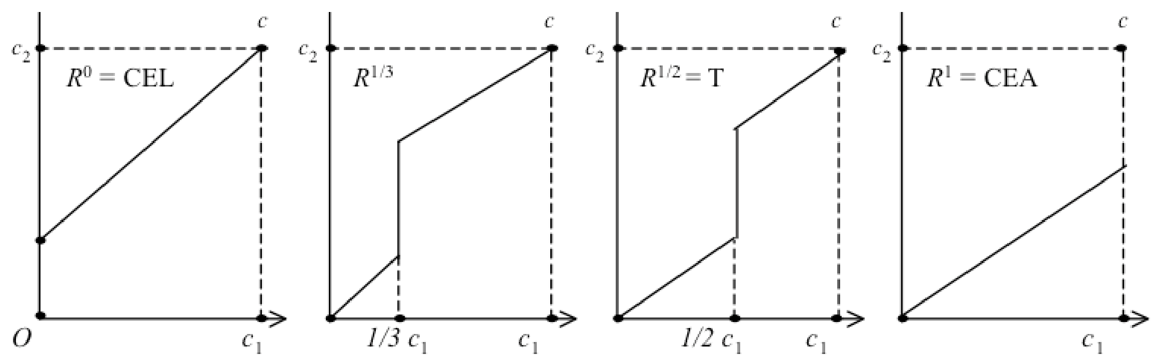

Figure 2.3. The TAL-Family of Rules

Also the Constrained Equal Awards rule is to be found in the Talmud, namely as the 'Rambam' method or the Jewish laws of lending and borrowing. It has also been presented as a mechanism to provide solutions for auctions (Kitrossky, 2001). Furthermore, according to Dagan (1996), most rabbinical authorities did adopt CEA as their law.

2.4 Vessels as a Hydraulic Representation

Finally, a non-mathematical way to depict a Bankruptcy Problem and derive a solution that is consistent with the allocation mechanism behind the Talmudic table was proposed by Kaminski (2000). In this model, claims are represented by vessels consisting of an upper and lower half, connected by links of infinitely small volume. The total system resembles a network of vessels where a volume of liquid substance – equal to the amount of the estate – can flow through until it reaches its balance. At that point, the allocations to the claimants can be deduced. It should be noted though that although the specific model described here coincides with the Widow Problem, there are different allocation rules that have a hydraulic representation (i.e. ones that are continuous, symmetric and consistent – see chapter 4). Examples of the three most common rules are presented in figure 2.4.b.

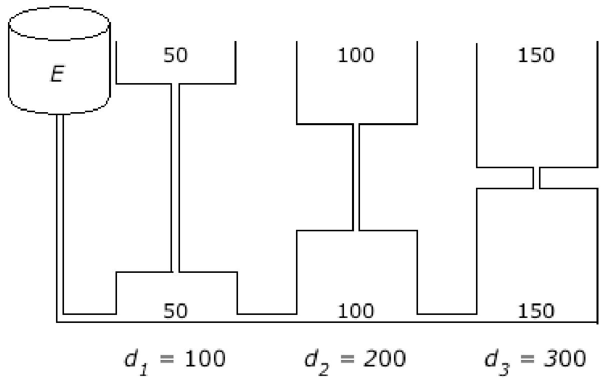

Figure 2.4.a. Kaminski's "Hydraulic" Representation of the n-Person CG Solution to the Widow Problem

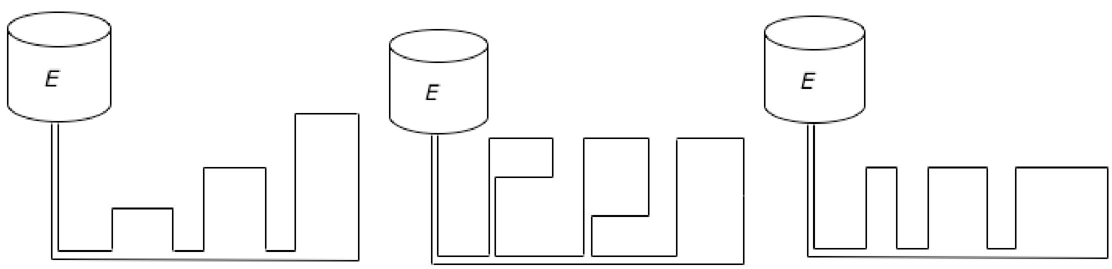

Figure 2.4.b. Kaminski's "Hydraulic" Representation of CEA, CEL and Proportional Division

Crucial to this hydraulic model of the Widow Problem is the symmetry between upper and lower vessels (and mutually between each of the vessels), in addition to the order preservation concerning claim amounts and losses of the claimants. As can be seen in the two exceptional situations to the standard Contested Garment principle (2.2.3 – An *n*-Person Solution), in case of small estates there is an equal division of the 'gain' amongst claimants, whereas in case of a large amount the algorithm makes sure that 'losses (i.e., $d_i - x_i$) are divided equally.

This self-duality property is also – visibly – present in Kaminski's model of the Bankruptcy Problem, making his system consistent with the *n*-person solution that is based on the allocation rule used in the Widow Problem. When small amounts of liquid are poured into the system, gains are shared equally amongst the claimants, yet as soon as the first threshold is reached for party one, the remaining liquids are divided amongst the remaining claimants, again until the level yields the next threshold. An opposite process is performed as soon as the liquid level reaches the upper parts of the vessels: now, *losses* are shared equally. This hydraulic representation can therefore be used to interpret the relation between the Contested Garment principle and the rules of Constrained Equal Awards and Losses, as explained in the previous section. Rationing from no estate to a certain threshold, the Constrained Equal Awards rules is used, whereas the losses-version is followed in cases of a relatively large estate.

2.5 A Note on Justice

In the present chapter several allocation rules have been discussed, focusing on finding the division method that would unravel the Talmudic Widow Problem. However, precedent to adoption in a real-life situation it will become clear that not all claimants would select the same allocation rule for their specific problem.

For instance, in applying the Contested Garment principle to a 2-person situation, claiming more than what is available is interpreted as claiming the entire estate. Here one can ask him- or herself if this property is fair; in case of bankruptcy, the claim of a party with an original deposit of 2 million compared to a party that provided only 1 million will be reduced to 1.5 million if that is the value of the estate left to divide (e.g., the worth of the real estate). Thus, the former party will feel like losing 500,000 no matter what, whereas the claim of the latter party is not reduced. One the other hand though it should be noted that both parties will end up losing fifty percent of their initial claim, which could again be interpreted as a fair allocation.

All in all, parties in a Bankruptcy Problem perceive pros and cons to specific rules, depending on the situation (viz. their claims, the claims of other parties and the amount of the estate). If claimants are rational, preference of some rule will depend on the properties of the allocation algorithms. In chapter 4 these properties will be elaborated upon, yet first two more solution concepts need to be explained, namely in the context of cooperative game theory.

Chapter 3

A Cooperative Game Theory Approach

By using a coalitional procedure to arrive at the *n*-person solution to the Widow Problem as in the previous chapter, the problem-solving approach came close to a game theoretic one. In order to apply some new allocation rules to the Bankruptcy Problem, it is necessary to rewrite the problem as a cooperative game in which parties are not considered purely as individuals but as players gathered in coalitions with a joint claim. Then, the nucleolus (Schmeidler, 1969) and the Shapley value (Shapley, 1953) will be presented as two unique one-point solutions to cooperative games.

3.1 Bankruptcy as a Cooperative Game

Grasping back to the formal definition of a Bankruptcy Problem in the second chapter of this paper, the following definition can be adopted for a cooperative game:

> In cooperative game theory, a game is denoted as the tuple <*N*, *v*>, where $N = \{1,2,...,n\}$ is the set of players and $S \subseteq N$ denotes a coalition S where N represents the grand coalition. In total there are thus $2^{|N|}$ coalitions that can be formed. The characteristic function $v : N \mapsto \Re$ denotes the value or worth of coalition S under the assumption $v(\emptyset) = 0$. The final allocations are given as a vector $x = (x_1, x_2, ..., x_n)$ where x_i represents the allocation to player *i*.

In a cooperative game, the value of the grand coalition N thus has to be divided amongst the players that participate in various coalitions with certain values. Taking the Talmudic Widow Problem as an example, interpreted as three Bankruptcy Problems (100 | 100, 200, 300), (200 | 100, 200, 300) and (300 | 100, 200, 300), the problem can be denoted as three separate cooperative games (N, v_i):

S	$\emptyset$	{1}	{2}	{3}	{1,2}	{1,3}	{2,3}	{1,2,3}
$v_1(S)$	0	0	0	0	0	0	0	100
$v_2(S)$	0	0	0	0	0	0	100	200
$v_3(S)$	0	0	0	0	0	100	200	300

Table 3.1. The Widow Problem as a Cooperative Game

The way a Bankruptcy Problem is rewritten into a cooperative game (from now on this will be called 'Bankruptcy Game') is a very important aspect and can be described as follows. The value for each coalition S is determined by taking the value that remains if the claimants that are not in the coalition (i.e. $N \backslash S$) first get their claims. Consequently it can be seen from the table that all coalitions are equally strong in the first case, that players 2 and 3 are equally strong in the second and that in the last situation player 3 is stronger than player 2, who in turn is stronger than player 1.

Thus, the original (bankruptcy) claims are now represented by the value of coalitions in a manner explained above; with this crucial step a Bankruptcy Problem is re-written into a cooperative game. Finally, in formal notation a cooperative game relates to a Bankruptcy Problem through $v(S) := (E - d(N \setminus S))_+$.

3.2 The Core

In any solution to a cooperative game, the contributions of each player to the coalitions are considered. An allocation is *individually rational* if the allocation exceeds or equals the worth of a coalition, $x_i \geq v(i)$ and *coalitionally rational* if $x(S) = \sum_{i \in S} x_i \geq v(S) \;\; \forall \; S$, i.e. all coalitions get an amount larger than or equal to their value. The core represents the collection of all coalitionally rational allocations possible and if no such solution exists, the core is empty. Furthermore, the core satisfies the *efficiency* property, recall: $\sum_{i \in N} x_i = v(N)$. Formally the core of a game <*N,v*> is denoted the collection $C(v) := \{x \in I(v) \mid \sum_{i \in S} x_i \geq v(S) \;\; \forall \; S \in 2^N \setminus \{\varnothing\}\}$ where *I*(v) is the so-called imputation set (i.e. those allocation vectors *x* that are both individually rational and efficient).

In Euclidian space $\Re^n$ (*n* players), a solution of allocations is a vector with the entries summing up to the worth of the grand coalition. Therefore a visual representation to obtain the extreme points of a (non-empty) core for a situation of three claimants could be the following:

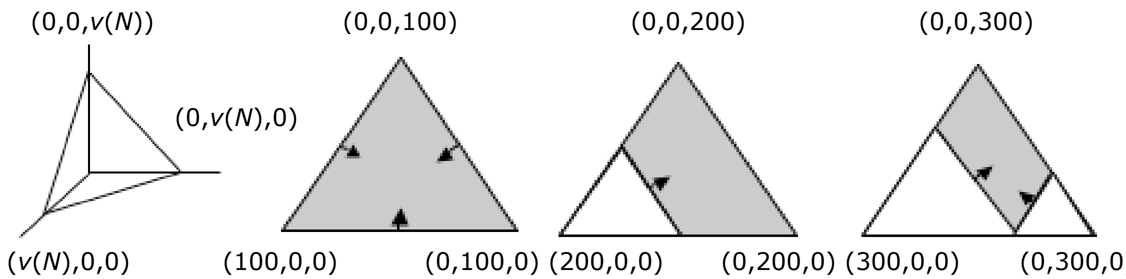

Figure 3.2. The Core of a 3-Player Cooperative Game

Here it should be noted that it could be seen as more valuable to derive at only one solution instead of a core full of possible allocations where further negotiations or different rules should be implemented before a final allocation can be determined. Moreover, in case the core of a Bankruptcy Game is empty another solution should be sought after. Furthermore, whereas for 3 players or claimants it is intuitive to visually represent the core and work with the approach showed above, for more dimensions it is not that simple (yet still possible). Fortunately, many unique one-point solutions exist of which the most common two will be discussed. These solutions exist not matter whether the core of the game is empty and being one-point solutions, they do not require further bargaining to come at a definite decision of how to allocate the estate.

3.3 The Nucleolus

The nucleolus, a concept developed by Schmeidler (1969), presents a unique point that minimizes the maximum excess of each coalition. The process that leads to this solution is as follows. Starting with a non-empty core, the values of each coalition except ∅ and *N* will be increased evenly until the point where proceeding would lead to an empty core. Should this procedure converge to a single point, the nucleolus has been reached. Otherwise, it is still possible to further increase the values of some coalitions where it doesn't infer an empty core, again until a single point remains. If the core is non-empty, the nucleolus will thus always be in the core. Similarly, when the original situation is an empty core the values of the coalitions are changed by equal amounts; in this case however they should be decreased until the core is non-empty. If this procedure doesn't result in a single point, the values of those coalitions can be increased in such a manner that the core doesn't become empty again, just as in the former case. In other words, this procedure constructs a non-empty core nonetheless by adjusting (certain) coalitions' values.

Thus, if the core is non-empty, coalitions could be allocated more of the estate until this increase would result in an empty core. Since every coalition prefers a minimal excess (e), where $e(S,x) = v(S) - x(S)$, the eventual solution will be the one in which the maximum excess is minimal. In case there are multiple allocations for which the maximum excess is minimal, instead of this maximum the second-high excess will be minimized and so forth for further overlapping excesses (the so-called lexicographic minimum).

Although the concept of the nucleolus was developed by Schmeidler (1969), Aumann and Maschler (1985) discovered the match between the table of the Widow Problem in the Talmud and the value of this nucleolus. Apparently, the nucleolus corresponds to the n-person solution based on the Consistent Garment principle, which can be said is a surprising result as the two allocation mechanisms are structurally different. Thus, by rewriting this specific Bankruptcy Problem into a cooperative game and computing the nucleolus, the Talmudic problem can be solved.

3.4 The Shapley Value

The Shapley value $\phi(N,v)$ by Shapley (1953) is a second unique one-point solution to a Bankruptcy Problem, obtained by taking the average of the marginal contributions of a player to the value of each possible coalition. The marginal contribution is defined as $v(S \cup \{i\}) - v(S)$; in other words, the allocation is dependent on how much value each claimant adds to the other claimants. For each possible order of step-by-step formation of the grand coalition (i.e., $n!$ different orders), the marginal contribution of the new entrant is stored. Finally, the average of these contributions for each individual player determines the allocation this player will receive, namely the following amount:

$$\phi(N,v) = \sum_{S \subset N \setminus \{i\}} \frac{s!((n-s-1)!}{n!}(v(S \cup \{i\}) - v(S)), \text{ where } s \text{ is } |S|.$$

Although this one-point solution does not coincide with the values from the Talmud, something that the nucleolus indeed does, the Shapley value does provide a useful alternative solution. In fact, O'Neill (1982) proposed the Shapley value as a way to give a bargaining solution for a simple claims problem. The Shapley value can be calculated for any cooperative game and is independent from the values of the core, i.e. it is irrelevant to the Shapley formula whether the final value corresponds to an individually or coalitionally rational solution.

Chapter 4

What is Fair? An Analysis of Properties

Thus far some allocation rules or algorithms have been presented, all following various kinds of fairness of justice principles. Throughout the description of the models remarks have been made questioning the 'fairness' of these models. Since a different set of properties comes with each rule, the fairness issue can be reduced to the question of "*what properties does one propagate?*" What the best set of properties is will depend on the situation in which the Bankruptcy Problem occurs and what the specific details of the problem are. Based on this analysis, the choice of which properties to embrace or to exclude will be left to the parties involved. Needless to say, the assumption is made that the claims of the parties are proper amounts, something that may be difficult to decide upon in certain situations. However, this matter is beyond the scope of this thesis. The selection of properties in this chapter is based on the descriptions of Dagan (1996), Herrero and Villar (2001), Moreno-Ternero and Villar (2006) and Thomson (2003).

An Inventory of Properties

In the following, an overview of several relevant properties will be given, some of which are already mentioned before.

The following properties are not included in table 4 as they are basic properties that are taken as self-evident. First of all, ***feasibility*** in the context of a Bankruptcy Problem implies that a total allocation cannot be larger than the estate ($\sum_{i \in N} x_i \leq E$), whereas according to ***efficiency*** the total allocation should be at least the size of the estate ($\sum_{i \in N} x_i \geq E$). The allocation to each claimant and each claim amount should also be ***non-negative*** in any Bankruptcy Problem ($x_i, d_i \geq 0$) and furthermore the property ***claim boundedness*** applies, i.e. no party should be allocated more than its claim ($x_i(N,E,d) \leq d_i$).

Although the next six properties are satisfied by all common allocation mechanisms discussed in the previous chapters, they give valuable information about the meaning of the rules.

Symmetry or ***equal treatment of equals***: parties with same claims should also be allocated the same amount. For $i, j \in N$, $d_i = d_j \Rightarrow x_i(N,E,d) = x_j(N,E,d)$.

Continuity: if small adjustments are made in the parameter values of a Bankruptcy Problem, the allocations should not change by much. If (E', d') converges to (E, d) then $x(N, E', d')$ converges to $x(N, E, d)$.

Order preservation (Aumann & Maschler, 1985): claimants claiming more should receive more and similarly lose more than others. $d_i \leq d_j \Rightarrow (x_i(N,E,d) \leq x_j(N,E,d), d_i - x_i \leq d_j - x_j)$. If an allocation rule satisfies this property, it will also be symmetric.

Homogeneity or ***scale invariance***: if the estate and the claim amounts are multiplied by a constant, allocations should also be multiplied by this constant. $x(N, mE, md) = mx(N, E, d)$.

Monotonicity: as mentioned in Moreno-Ternero and Villar (2006), there exist multiple kinds of monotonicity, also called solidarity properties. In *resource or net worth monotonicity* (Curiel et al., 1987) the allocation of each claimant should not be smaller in a situation where the estate is larger (ceteris paribus). For $i \in N$, $E' > E \Rightarrow x_i' \geq x_i$. The same idea is present in the property *claims monotonicity*, where a claimants' allocation should not be smaller when his/her claim amount is increased, ceteris paribus. For $i \in N$, $d_i \leq d_i' \Rightarrow x_i \leq x_i'$.

According to the property ***self-duality*** (Aumann & Maschler, 1985) gains and losses are treated in a similar way, viz. $x(N,E,d) = d - x(N,\sum_i d_i - E,d)$.

The following set of properties concern the structure that is inherent to them of how claims and claimants are treated (Herrero & Villar, 2001):

Consistency: taking a sub-section of the entire group of claimants, the allocation to this sample should be the same as in the situation where the complete group of claimants is taken into consideration. For all $S \subset N$ and $i \in S$, $x_i(N,E,d) = x_i(S,E_S,d_S)$ where $E_S = \Sigma_{i \in S} x_i(N,E,d)$ and $d_S = (d_i)_{i \in S}$.

Independence of (or ***invariance under) claims truncation, independence of irrelevant claims*** (Curiel et al., 1987): if claims exceed the estate, the excess of the claims should be neglected and thus should the new claim considered (d_i') be equal to the value of the estate: $d_i' = \min\{E, d_i\}$. The allocations of rules that satisfy this property will be the same no matter whether the claims exceed the estate or not, as the name of this property already implies.

Composition (up) from minimal rights or ***v-separability*** (Curiel et al., 1987): if there is a positive amount left after the subtraction of the sum of claims of other claimants *N\i* from the estate, this amounts to the minimum right (or uncontested amount) of claimant *i*. Thus, *minimum right* = $\max\{0, E - \Sigma_{j \in N \setminus \{i\}} d_j\}$. This property is satisfied if the allocations are the same no matter whether this minimum right is taken into consideration. If a rule satisfies *efficiency*, *claim boundedness* and *non-negativity* or if it is both *self-dual* and *independent of claims truncation*, it follows that this minimal rights property also satisfies the minimal rights property.

Securement (Moreno-Ternero & Villar, 2004): each claimant should be allocated a minimum part of $\frac{1}{n}$th of its claim (i.e., if the claim does not exceed the estate; otherwise $\frac{1}{n}$th of the estate, as under *claims truncation*), regardless of the claims of other parties, which was the case under the previous property. Thus, the property is satisfied if $x_i(N,E,d) \geq \frac{1}{n}\min\{d_i, E\}$.

Furthermore, these properties concern changes in (the composition of) the estate:

Resource additivity: if an estate is split into parts, the allocation on the entire estate should equal the sum of allocations on the parts separately. $E = E_1 + E_2 \Rightarrow x(N,E,d) = x(N,E_1,d) + x(N,E_2,d)$.

Composition up (Young, 1988): if the estate turns out to be larger after the allocation of the initial amount, no matter whether a new allocation is determined based on the new estate or if the allocation of the additional amount (on the remaining claims) is added to the first allocation, the final allocation should be the same. $x(N,E_1+E_2,d)=x(N,E_1,d)+x(N,E_2,d-x(N,E_1,d))$. Under ***composition down*** or ***path independence*** (Moulin, 1987) the same idea is applied, only in case the estate turns out to be smaller. *Composition* in general is satisfied if, after the estate is split, the final allocation can be computed by adding the allocation of the split amounts, where the second allocation is computed over the remainder of claims.

The following two (types of) properties are so-called *protective properties* as these consider an unequal distribution of claims, i.e. cases where very small or large claim amounts are present amongst the parties (Moreno-Ternero & Villar, 2006):

Exemption: if a claim is small, this claimant should receive the entire claim, where a claim amount is 'small' if it is smaller than in equal division of the estate. For all $i \in N$, $d_i \leq \frac{E}{n} \Rightarrow x_i = d_i$.

Sustainability or ***conditional full compensation***: if a claim is small, this claimant should receive the entire claim amount, where a claim is defined 'small' if – when the claims of all parties with larger claims are reduced to this amount – enough of the estate would remain to fulfill everyone's (reduced) claim. For all $i \in N$, $\Sigma_{j \in N} \min\{d_i,d_j\} \leq E \Rightarrow x_i = d_i$. If an allocation rule is *sustainable*, it also satisfies the *exemption* property. Furthermore, if both *exemption* and *consistency* are satisfied, the rule is also *sustainable*.

Exclusion: in this dual property of exemption if a claim is small this claimant should not receive any part of the entire claim, where a claim amount is defined `as 'small' if it is smaller than the average loss: $d_i \leq \frac{D-E}{n} \Rightarrow x_i = 0$, where $D = \sum_i d_i$.

Independence of residual claims: in this dual property of sustainability, parties with large claims should have the first priority in being rewarded over parties with small(er) claims. Here the residual claim of a party *i* is the sum of excess values, where the excesses are determined by taking the difference between other party's substituted claims (the maximum of zero or d_j minus *i*'s claim) and the estate. If the claim is residual, the party receives nothing. $E \leq \Sigma_{j \in N} \max\{0, d_j - d_i\} \Rightarrow x_i = 0$. Again, if *independence of residual claims* is satisfied, then *exclusion* is satisfied as well.

Finally, the following two properties state that no benefits should result from a manipulation in the set-up of claims amongst claimants:

No advantageous reallocation or ***transfer*** (Moulin, 1985): claimants cannot gain by redistributing their claims amongst each other. For $M \subset N$, $\Sigma_M d_i = \Sigma_M d_i' \Rightarrow \Sigma_M x_i(N,E,d) = \Sigma_M x_i(N,E,(d_i')_{i \in M}, d_{N \setminus M})$.

No advantageous merging or splitting, ***non-manipulability*** or ***strategy-proofness*** (O'Neill, 1982): no party should gain more by splitting its claim as being several claimants, nor should several claimants gain more by merging their claims. For $S \subset N$, $i \in S \Rightarrow x_k(E,\tilde{d}) = \sum_{i=k}^{n} x_i(E,d)$ where $\tilde{d} = (d_1, d_2, ..., \tilde{d}_k)$ and $\tilde{d}_k = d_k + d_{k+1} + ... + d_n$.

Table 4 provides an overview of which properties are satisfied by the common allocation rules CEA, CEL, the *n*-person Contested Garment procedure, the proportional rule and the game-theoretic results of the nucleolus and Shapley value. Recall that the so-called TAL-family of rules (Moreno-Ternero & Villar, 2006) as discussed in chapter 2.3 comprises CEA, CEL and the *n*-person Contested Garment procedure by setting the θ-values to respectively 1, 0 and ½. This notation is particularly useful in this context as by changing parameter θ, one can also change the set of properties that is satisfied.

	TAL-RULES	PROPORTIONAL RULE	NUCLEOLUS ($R^{1/2} = T$)	SHAPLEY VALUE
Symmetry	$R^{\theta} \forall \theta \in [0,1]$	Yes	Yes	Yes
Continuity	$R^{\theta} \forall \theta \in [0,1]$	Yes	Yes	Yes
Order Preservation	$R^{\theta} \forall \theta \in [0,1]$	Yes	Yes	Yes
Homogeneity (Scale Invariance)	$R^{\theta} \forall \theta \in [0,1]$	Yes	Yes	Yes
Resource Monotonicity	$R^{\theta} \forall \theta \in [0,1]$	Yes	Yes	Yes
Claims Monotonicity	$R^{\theta} \forall \theta \in [0,1]$	Yes	Yes	Yes
Self-Duality	$R^{\frac{1}{2}} = T$	Yes	Yes	Yes
Consistency	$R^{\theta} \forall \theta \in [0,1]$	Yes	Yes	No
Independence of Claims Truncation	$R^{\theta} \forall \theta \in [\frac{1}{2},1]$	No	Yes	Yes
Composition (up) from Minimal Rights	$R^{\theta} \forall \theta \in [0,\frac{1}{2}]$	No	Yes	Yes
Securement	$R^{\theta} \forall \theta \in [\frac{1}{2},1]$	Yes	Yes	Yes
Dual of Securement	$R^{\theta} \forall \theta \in [0,\frac{1}{2}]$	Yes	Yes	Yes
(Resource) Additivity	No	Yes	No	No
Composition (Up)	$R^{0} = CEL \quad R^{1} = CEA$	Yes	No	No
Composition Down (Path Independence)	$R^{0} = CEL \quad R^{1} = CEA$	Yes	No	No
Sustainability	$R^{1} = CEA$	No	No	No
Exemption	$R^{1} = CEA$	No	No	No
Independence of residual Claims	$R^{0} = CEL$	No	No	No
Exclusion	$R^{0} = CEL$	No	No	No
No Advantageous Reallocation	No	Yes	No	No

Table 4. An Overview of Properties for Several Common Allocation Rules

From this table it can be seen clearly which rules satisfy the properties one regards as relevant in a particular situation. For instance, CEA is the unique rule that satisfies the properties *independence of irrelevant claims, composition* and *equal treatment or symmetry* (Dagan, 1996). Another collection of the properties *scale invariance, path-independence, symmetry, composition* and *consistency* is uniquely satisfied by both CEA and CEL (Herrero & Villar, 2001). It is also interesting to see that the proportional, CEA and CEL rule can all be seen as egalitarian (due to *symmetry*), yet the parameters on which equal assumptions are made differ, i.e. equal ratio's, awards or losses. Furthermore, looking at the claimants' allocations, CEL (CEA) gives a priority in allocating to parties with larger (smaller) claims, whereas no such distinction is made in a proportional allocation (Herrero & Villar, 2001).

It should be realized that the two rightmost columns of table 4 present solutions to cooperative games, as opposed to the other allocation mechanisms that do not. Therefore, looking to cooperative games in general the list of properties could be extended with properties that specifically apply to the nucleolus and Shapley value. Such properties are the earlier mentioned *individual* and *coalitional rationality*, *anonymity*, *additivity* and the *dummy property* that will not further be touched upon in this paper. However, in the context of a Bankruptcy Game, many of these properties do not apply anymore. This is due to the fact that a Bankruptcy Game is a convex game, meaning that function $v(S)$ is *supermodular*: $\forall S,T \subseteq N \Rightarrow v(S \cup T) + v(S \cap T) \geq v(s) + v(T)$, a property that will return in chapter 6.2. Another result of applying solutions of cooperative games to the specific class of Bankruptcy Problems is that under this specific situation of a convex game, certain properties *will* now indeed be satisfied. For instance, due to the convexity of the Bankruptcy Game its core is nonempty and therefore, next to the nucleolus, also the Shapley value will be found within the core.

Discussion

Although table 4 provides a good basis for comparison of the allocation rules, based on this overview no conclusion can be drawn regarding which rule would be the most favorable in many situations. Instead, in each particular context of the Bankruptcy Problem it should be determined which allocation rule best covers the properties one likes to see.

To discuss a few properties, *sustainability* and *exemption* clearly are no just attributes in all situations. For instance in a case where the estate represents a total amount of taxes to be paid and claims represent the wealth of the parties, it could be seen as fair to exclude people with a very small income to the tax payment and let higher-income parties cover more of the taxes. In that case, the dual properties of *independence of residual claims* or *exclusion* would be fairer to apply. A similar intuitive description of a situation in which these last two properties can indeed be seen as just is a health care system that does not provide coverage of the cost of a small illness, but does reimburse the claims of people with a serious disease. The small claims are now regarded as irrelevant in a Bankruptcy Problem where the estate can be interpreted as an important need. On the other hand, the property of *exemption* could imply that parties with small claims (i.e., smaller than E/n) should be allocated the amount of their claim for sure, since if all other claimants would have a claim of (at most) this size no Bankruptcy Problem would exist in the first place (Herrero & Villar, 2001). Carrying these results further to possible allocation rules, based on *independence of residual claims* and *exclusion*, CEL is a desirable rule, as is CEA for the duals of these attributes.

Another issue is present in the property *independence of claims truncation*. According to Dagan (1996, p. 59) it can be said that 'The set of allocations in a bankruptcy problem is the same as in the problem where the claims are reduced to the relevant claims'. This implies that the property of independence of claims truncation would be a fair and feasible property for any allocation rule. However, note that the proportional rule does not satisfy this property. Therefore this allocation rule would not be considered appropriate, although it can be seen as an important aspect that no matter the height of one's claim, his or her allocation will be proportional to this claim and not to only a part of this claim.

Furthermore, the property *composition* could be seen as an appropriate quality in a situation where the final allocation should be the same no matter there is step-wise clearance of a business on the edge of bankruptcy or whether the firm went bankrupt at once. In this case the estate of the bankrupt firm is assumed to exist of different types of property, where after dividing the first type (e.g., real estate) a second type (e.g., furniture) is to be divided (Herrero & Villar, 2001).

However, one should be careful in applying a selection of just and feasible properties to a real-life situation and expecting an allocation rule that logically follows. In a survey by Gächter and Riedl (2004), normative judgments of participants on common decision rules were compared with a ranking they gave in an actual problem situation. The research showed that normative judgments people have concerning what rules or properties of rules are fair did not correspond to their actual behavior in case they were in a bankruptcy situation. Whereas the proportional rule was regarded as the most just (as opposed to equal division) regardless of the claims and CEA and CEL were found appropriate in situations were the claims had respectively a very or little asymmetric distribution, being a claimant themselves, the participants chose the CEA rule. An explanation of this outcome could be that in a real situation with a known estate, (high) claims will now be infeasible. Thus, regardless of how skewed the claims distributions is, a safe choice is to choose the CEA rule that gives an allocation in between the proportional and equal division rule.

Likewise, it is surprising to see that a property that is perceived as fair by many, viz. *symmetry* or *equal treatment of equals* is often not satisfied in real-life situations (Thompson, 2003). Although a complete and correctly defined Bankruptcy Problem should consist of claimants for which the only relevant differences are incorporated in the height of claims and that are indeed equal in all remaining relevant aspects, this assumption is apparently violated in many cases. Thus, if there are aspects distinguishing claimants that are not yet incorporated in the claims, no fair allocation can be made if *symmetry* is (incorrectly) assumed. A solution could be to allocate different weights or more complex parameters to claimants that cover the remaining differences between them.

Finally, another interesting research conducted by Bosmans and Schokkaert (2007) shows the importance of considering the specific situation in which the Bankruptcy Problem occurs. In their experiment, it was compared which division rules were perceived as the most appropriate in both a division of revenue amongst contributors to a business firm and an allocation of tax revenues to pensioners with different historical contributions. Although both situations follow a similar structure, in the latter situation more egalitarian division rules were proposed as basic needs were weighted more heavily than 'wants' in the business context, yet overall the proportional rule was the most popular rule. Furthermore, the properties of (*strict*) *order preservation* and *nonzero allocations* were seen as specifically important.

Part II
Application to the EU Fisheries Management

Fisheries management has acquired a major position in worldwide discussions regarding reform, based on decreasing quality of the ecological environment in global waters. This makes possible changes in management systems an important and interesting issue, having to deal with various complexities. In this second part of the present paper the fisheries management system within the European Union will be focused on, as one of the areas where the Bankruptcy Problem and related allocation mechanisms show their applicability. In chapter 5 a short overview will be given regarding the fishing situation to set the context of the problems, together with a more technical analysis of the management system the European Union has adopted in dealing with fisheries. Finally, two scenarios of real-life problems will be considered in chapter 6, i.e. the use of an allocation model that provides a relatively good road to sustainability in fish stocks and the usage of the Bankruptcy Problem in how to deal with new countries or fisheries that want to join the existing division scheme of fish quota.

Chapter 5

Introduction to the Global Fishing Situation

In the following chapter an outline will be provided of the current state in the global fish(ing) situation. Trends of over-fishing and imbalanced fish stocks will be touched upon, based on the thoughts of biologist Pauly (Pauly & Watson, 2003). In the second paragraph the underlying reason of inefficiency related to common property will be explained, following Arnasons (2007) research of the fishing industry.

5.1 Ecological Trends in Global Waters

The current state of fish ecology is an issue of paramount importance. In a time of a rising awareness of both environmental trends and the importance of fish consumption in a healthy diet, fish seems to have become a resource that is subject to an increasing consumer demand and at the same time is supported in its preservation by a growing body of people. Not surprisingly, the topic has become a frequently returning focus of amongst others newspapers, environmentalist groups and scientific research.

To outline the urgent call of reform by biologist Pauly (Pauly & Watson, 2003), he discovered that the global fish catch is decreasing even more than official numbers indicate while fishing capacities increase, which means that fish is becoming more and more scarce. The reason why consumer demand is kept fulfilled is that approximately fifty percent of the fish is caught on different continents. This is financially feasible for the fisheries because their work is supported by subsidies, as Pauly argues. Due to this – globally adopted – system of subsidies fish is being caught that would not be profitable otherwise, undermining sustainable fish stocks. More concretely, Pauly states that of the 20.000 fish stocks more than half is already over-fished and a quarter has even collapsed. At the same time the recovering process in protected sea areas improves by only an increase in biomass of five percent a year.

Looking at the balance amongst fish stocks, one can define a major problem that influences the ecological sustainability. Named 'fishing down the food web' (Pauly at al., 1998) the following trend is described. While at first the physically larger fish types have been over-fished, fisheries turn more and more to smaller species, as the larger fishes are getting distinct. This causes a trend of moving closer and closer to the lower levels of the food chain for fish, which is an unsustainable process, in extreme scenario's leading eventually to a sea with only jellyfish and plankton. Next to that local fisheries (i.e. the fisheries that generally catch larger species with their more selective fishing equipment) get overshadowed by heavy-capacity boats such as deep-water trawlers. While this supports the 'fishing down the food web', these ships also cause damage to the ocean floors by scraping over the floor with nets (Pauly & Watson, 2003).

5.2 Fisheries from an Economic Perspective

When looking at the current state of fish stocks, one should also consider the economic structure inherent to the field of fisheries. As with any common property, the fisheries industry suffers problems of inefficiency. In order to prohibit fisheries from freely operating - which would lead to unbalanced environments and heavy over-fishing – social institutions are initiated to control the fishing efforts. However, much of the fishing activity nowadays still occurs under the conception of universal property and the institutions that do exist are ill-equipped in governing the actual activities, as Arnason (2007) argues. Also, management in itself should not get so much focus that it overshadows the actual fishing efforts, as is a substantial problem in the industry. Arnason notes that the global profit loss due to subsidies provided to fisheries is estimated to be around fifty percent of the actual landed value of fish. Thus, except from the already mentioned problems over overexploitation of the fish stocks and a skewed or unhealthy ecological state of seas, economically seen there are additional inefficiencies. Also mentioned are the insignificant or low profits made by individual fisheries, low incomes in the fishing industry and a low overall contribution to the Gross National Product (GNP) (Arnason, 2007).

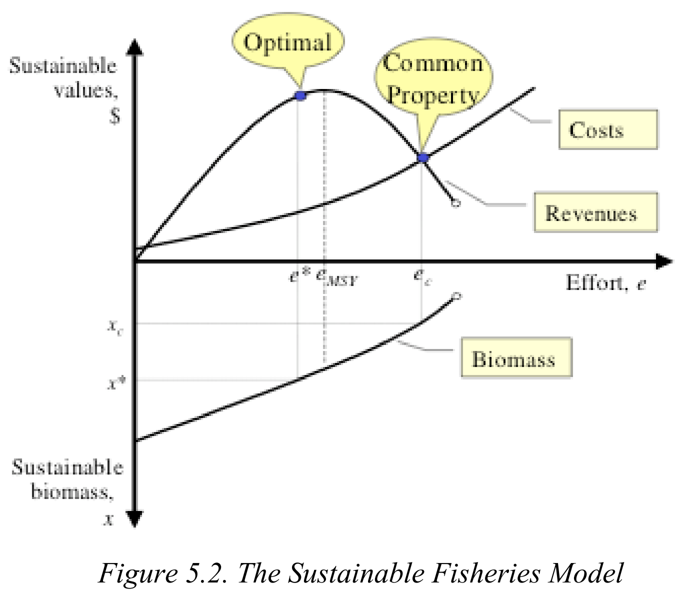

Figure 5.2. The Sustainable Fisheries Model

Figure 5.2 (Arnason, 2007, p.2) provides a summary of sustainable profits and biomass values for different fishing efforts. It should be noted that the biomass gets smaller when coming closer to the x-axis; after a certain threshold, fish stocks cannot recover anymore and collapse. Furthermore, if the fishing efforts remain stable (ceteris paribus), the related values of revenue and biomass will be sustainable, i.e. they coincide with the long-term average of these values. Now, looking at the two situations pointed out in the figure, at effort e^* the profit of the fishery is maximized under an ecologically sustainable stock size, slightly below the revenue for e_{MSY} (maximum sustainable yield). Therefore this 'profit maximizing sustainable fisheries policy' is desirable. However, an arrangement of common property would lead to e_C, at the intersection of cost and revenue. Not only does this situation decrease the biomass even further, also the contribution to the GNP is insignificant. This situation therefore follows the concept of 'the tragedy of the commons' where individuals cannot prevent this outcome from occurring. Rationally, each fishery will try to catch as much as possible from the existing biomass in order to gain some profit. Clearly, this situation is far from desirable; as a result of the difficult nature of fisheries, fisheries management is needed in order to arrive at a better outcome.

5.3 Management Structures in the EC Fisheries

Now that it has been made clear that management is of paramount importance in the fisheries sector, it is time to look into the management systems that are actually adopted. Characteristic of fisheries management is the dense network of rules and regulations and the corresponding sanctions, resulting in an ungraspable and confusing system that is easily (unintentionally) violated. Thus, complexity is inherent in the development of the management system; for each problem that arose, new and more detailed amendments were made. In a contingent environment of biological and environmental fluctuations, changes in fishing practices and in state-policies and their priorities, frequent revisions of and additions to the existing system are needed. As Jentoft and Mikalsen (2004, p. 127) point at, this 'vicious circle of fisheries management rule making' also brings with it an increasing cost of management and enforcement. Although Jentoft and Mikalsen only looked at the situation within Norway, as fisheries management is obviously existent at several institutional levels, the EU unsurprisingly faces a similar problem. The resulting situation of endless discussions concerning management reform is also supported by Pauly (Pauly & Watson, 2003), who points at the ongoing debates in the industry. Nevertheless, a judicial system is needed in order to protect the biosphere and at the same time the economical continuation of fisheries. Thus arises the question: how to arrive at and maintain an effective policy?

Management in the EU – The TAC

In order to manage the fisheries within the European Union, the European Commission (EC) Fisheries established the Common Fisheries Policy (CFP) in 1983, a program that has been subject to numerous adaptations to developments in the field, most recently in 2002. The EC Fisheries uses research and recommendations from independent boards such as the International Council for the Exploration of the Sea and the Directorate for Conservation Policy. Next to safeguarding the situation of both fish and fishing fleet within the EU, the commission also deals with areas that are only partially shared by member states in bilateral agreements. Yet another important part of its activities comprises the actual control and of regulations, although member-states have their own responsibility to enforce the rules of the CFP. However, in the line of what has been discussed previously, there are still many obscurities concerning control and sanctions due to the complex network of rules (European Commission, 2004).

An important concept to the management of European fisheries is the so-called Total Allowable Catch (TAC), an (annual) quota set by the EC Fisheries since 1976. This quota is determined for each area touching the member states – based on a division scheme of these surrounding seas – and each fish type, based on recommendations by research institutes. The TACs then need to be further allocated to the vessel groups within the member states (Karagiannakos, 1996).

The TAC-system is a branch of biological fisheries management according to Arnason (2007), aimed to safeguard fish stocks but not restoring the common property arrangement from an economic point of view, as discussed in the previous section 5.2. He argues that although the TACs at first reduce the fishing efforts, leading to a profitable situation, fisheries will subsequently increase their efforts in order to gain even more profit. Thus, the management system needs to provide incentives to the parties involved not to exceed their quota, i.e. complex punishment systems are needed to reduce this trend. Also biologist Pauly (Pauly & Watson, 2003) does not believe in the effectiveness of setting a TAC for each fish species, since fisheries in real-life catch multiple fish types simultaneously, making it more difficult to keep track of these quota. This is especially the case with less selective fishing techniques. Furthermore, he argues that the TACs in the EU have structurally been set too high, viz. at the level where collapsing is prevented though recovery still prohibited. The latter has been admitted by the European Council, which has been setting certain TACs at a higher level than suggested by the EC Fisheries (European Commission, 2001).

It can thus be seen that using a TAC-system is not without drawbacks. However, setting aside these critiques on the implementation of TACs by the EC Fisheries as it can be said that no good alternative is at hand either, it could be more fruitful to search for ways that *improve* the system. To give some examples, the reform of the CFP included the measures of – additionally to setting the TAC – limiting the number of days on which fishery activity is allowed, increasing the number of protected areas, setting restrictions on fishing equipment (e.g., the size of mazes in a net to make the gear more selective) and implementing minimal sizes for fish to be landed (European Commission, 2004).

Chapter 6

Bankruptcy Problems in EU Fisheries Management

In this chapter, two scenarios will be sketched in which the Bankruptcy Problem can prove its use. First, the formulation of a dynamic Bankruptcy Problem by Iñarra and Skonhoft (2008) will be discussed, in which the total catch is to be divided between the coastal fleet and deep-ocean trawlers. Proposed is to use the Constrained Equal Award rule in order to shorten the fish stocks' recovery period and arrive at a healthier biological balance on long term. The second part of the chapter will then show how the Bankruptcy Problem can be used to arrive at solutions of how to deal with new entrants in a certain area, i.e. member states joining the European Union.

6.1 Sustainability in the Fish Stocks

Taking the North East Atlantic Norwegian cod as an example, which is highly relevant as the North-sea cod is already on the verge of collapsing (European Commission, 2004), Iñarra and Skonhoft (2008) developed a dynamic Bankruptcy Problem to find the best model that would be able to improve the situation and reduce the recovering time for fish stocks using the TAC system. Based on these results, they advise to no longer use the proportional rule that is currently adopted to divide the TACs, where the height of the claims is determined on the basis of what has been caught in the past. Instead, CEA is proposed as a better alternative, since (1) it is perceived as an unfair assumption that fleets having caught more in past therefore deserve higher quota at present; (2) as will be shown in the remainder of this section, CEA can lead to both a larger steady-state stock size and TAC and (in certain cases) profits, plus a shorter time is needed to reach this steady-state.

A (Dynamic) Bankruptcy Problem

The situation can be formulated as a Bankruptcy Problem for the reason that the claims on fishing rights will exceed the 'estate' of allowable amounts of fish to catch. Clearly, in case of a significantly large supply of fish there would not be such a problem, however this state would imply the undesirable 'tragedy of the commons' situation as explained before. Therefore the estate of fish that can be caught is reduced to the TAC quota. Furthermore, following the way the TAC system is adopted by the EC Fisheries the claims or fishing rights can be based on the historical catch of each party. Further, there are several levels on which the problem can be seen; claimants can be for instance individual fisheries, groups within a nation, or nations with the EU. In the research of Iñarra & Skonhoft (2008), the trawler and coastal fleet are taken as two parties. As the cod fishery in the North-East Atlantic is more or less equally shared between Norway and Russia, where Russia uses trawlers exclusively as opposed to 70 percent in Norway, only the situation of the Norwegian fleet is considered. Here it is important to consider the difference in the nature of the claimants; whereas it is assumed that the selective coastal fleet does not influence the natural growth rate of the fish stock, trawlers do by both damaging the environment and catching immature fish.

All in all, this specific Bankruptcy Problem is denoted (*TAC, h*), where the TAC is the estate to be divided and h the claims vector of harvesting rights. The two allocation rules of the TAC that are the focus of the investigation, i.e. the proportional and CEA division rules are represented by respectively:

$q_i = PROP_i(TAC, h) = \frac{h_i}{\sum_{i=1}^{N} h_i} TAC$, and

$q_i = CEA_i(TAC, h) = \min\{h_i, \lambda\}$ with λ such that $\sum_{i=1}^{N} \min\{h_i, \lambda\} = TAC$ and where N denotes the number of harvesters and h_i the harvest by vessel group i.

Although this model is complete, a more pertinent representation of the situation is made by a *dynamic* Bankruptcy Problem. Even though a TAC regulating scheme has been designed for a certain period, it is necessary to update the system (yearly) with actual catches. Additionally, the different harvesting types (e.g., selectivity) and longer-term developments such as the recovery period of the fish stock should be taken into consideration as they also influence the subsequent stages of the 'bankruptcy process'.

The Fishery Problem in Formulas

Iñarra & Skonhoft (2008) developed the following set of equations that form the framework of their dynamic bankruptcy model. An overview of all parameter definitions is provided in appendix A.

Formula (1) represents the natural growth function of the fish biomass, not subject to regulation; this function will be step-wise adjusted until the final representation by equation (5). The presence of one of the allocation rules presented above leads to the second adaptation of the fish stock dynamics in (2) that now incorporates both the size of TAC_i and q_i that is adopted.

(1) $$X_{t+1} = X_t + F(X_t, h_1(X_t), ..., h_N(X_t)) - \sum_{i=1}^{N} h_i(X_t)$$

(2⇐1) $$X_{t+1} = X_t + F(X_t, q_{1,t}, ..., q_{N,t}) - TAC_t$$

Equation (3) shows the net intrinsic growth rate of the fish stock, which is heavily dependent on the selectivity of the fleet and again the allocation rule used to divide the harvest. Parameter γ_i denotes the selectivity, where $\gamma = 1$ means the vessel is perfectly selective such that there is no difference between the net and gross intrinsic growth rate. Furthermore, growth function (4) that is a component of the function for fish stock dynamics now incorporates the common logistic natural growth function where K denotes the carrying or natural capacity of the fish stock. Consequently, (2) is changed to (5), the final representation of the fish biomass dynamics.

(3) $$\tilde{r}_t = r + 1 - \sum_{i=1}^{N} \alpha_{i,t}\gamma_i \text{ where } \alpha_{i,t} = \frac{q_{i,t}}{TAC_i} \text{ and } \sum_{i=1}^{N} \alpha_{i,t} = 1;$$

(4) $$F(X_t, q_{1,t}, ..., q_{N,t}) = \tilde{r}_t X_t (1 - X_t / K);$$

(5⇐2) $$X_{t+1} = X_t + [r + 1 - \sum_{i=1}^{N} \alpha_{i,t}\gamma_i] X_t (1 - X_t / K) - b(X_t - X_{\min});$$

Now, combining this biomass function with that of the net intrinsic growth rate (3), the steady-state functions – presenting the long-term stable values, represented in the formula's by an asterisk – for the fish-stock size (6), intrinsic growth rate (7) and TAC (8) can be derived.

(6) $$X^* = \frac{K(\tilde{r}^* - b) + \sqrt{K^2(\tilde{r}^* - b)^2 + 4\tilde{r}^* bKX_{\min}}}{2\tilde{r}^*};$$

(7) $$\tilde{r}^* = r + 1 - \sum_{i=1}^{N} \frac{q_i^* \gamma_i}{b(X^* - X_{\min})};$$

(8) $$TAC^* = b(X^* - X_{\min});$$

Finally, function (9) is the harvest function of the vessel groups, incorporating parameters $E_{i,t}$ (the fishing effort or size of the vessel group) and θ_i (the productivity or 'catchability' of the vessel group). The profit function is given in (10), where parameters p_i and c_i denote the catch price and the unit operation costs of the vessels.

(9) $$q_{i.t} = \theta_i E_{i,t} X_i;$$

(10) $$\pi_{i.t} = (p_i - c_i) / \theta^i X_i) q_{i,t}$$

Going through some aspects that are rooted in the previous set of equations more elaborately, first of all the parameter b occurs in the final equation that represents the fish stock dynamics (5). This parameter results from the equation $TAC_t = \max[0, b(X_t - X_{\min})]$, viz. the amount of fish that should be caught under the proportional rule, equal to some proportion of the excess stock. Here, b denotes this particular fraction and $X_{\min}$ stands for the minimum threshold level used to determine the excess value. Now, because it is assumed that $X_t > X_{\min}$ and X_t is increasing, it shows that TAC_t is also increasing. Furthermore, high values for b will shorten the recovery period along with a smaller stock and TAC^*, whereas high values for $X_{\min}$ lead to the reverse.

Also, both parameters c and p are assumed to be in favor of the coastal fleet, meaning that the catch price is higher as a result of the size and quality of this fish caught and the unit cost is lower than that of the trawler fleet. Consequently, based on the last two functions it can be seen that in short-term, the more the vessels are allowed to catch, the more profit is yielded.

Finally, it should be clear that the "dynamic fishery bankruptcy model" gives a sequence of *TAC* as output. With the aim of restoring the fish stock as well as possible, TAC_t should be set low at the start of the process such that the catch does not interfere with the natural growth of the stock. The *TAC* can then be steadily expanded up till the steady-state values, i.e. the environmentally and economically sustainable situation that is pursued.

Outcomes

Based on real-life data, Iñarra and Skonhoft (2008) deduced the following findings. First of all, the crucial factor in their model is the selectivity of the fleet; selective fisheries like the coastal fleet are assumed not to have a negative impact on the natural growth function whereas the unselective trawlers do by harming smaller, immature fish. Following the traditional proportional division of the TAC, the trawler fleet would be allocated a relatively high proportion nonetheless, as a result of historical catches. By applying CEA though, the trawler fleet is allocated as least as possible while still considering their claims and more rights can be distributed to selective fisheries, therefore resulting in a higher natural growth of the fish stock. On long term this solution is also beneficial to the trawler fleet, as a larger steady-state fish stock brings with it a higher steady-state TAC. This conclusion also supports the advise of Pauly (Pauly & Watson, 2003) to decrease the trawling activity as much as possible in order to protect the fish stocks.

The graph in figure 6.1 (Iñarra & Skonhoft, 2007, p. 21) shows the development of the fish stock size using both methods, CEA and the proportional rule. Here it can be seen that a higher size of fish stock and TAC is reached under the steady-state condition. The increase in profit will be relatively large for the coastal fleet because of the larger TAC allocated to this party, however the profit of the trawler fleet can also improve in the new situation, as it results in an eventually higher TAC. Furthermore the recovery time, i.e. the time needed to arrive at the steady-state (*) situation is indeed shorter for CEA, as the impact of nonselective fisheries on the growth rate is reduced.

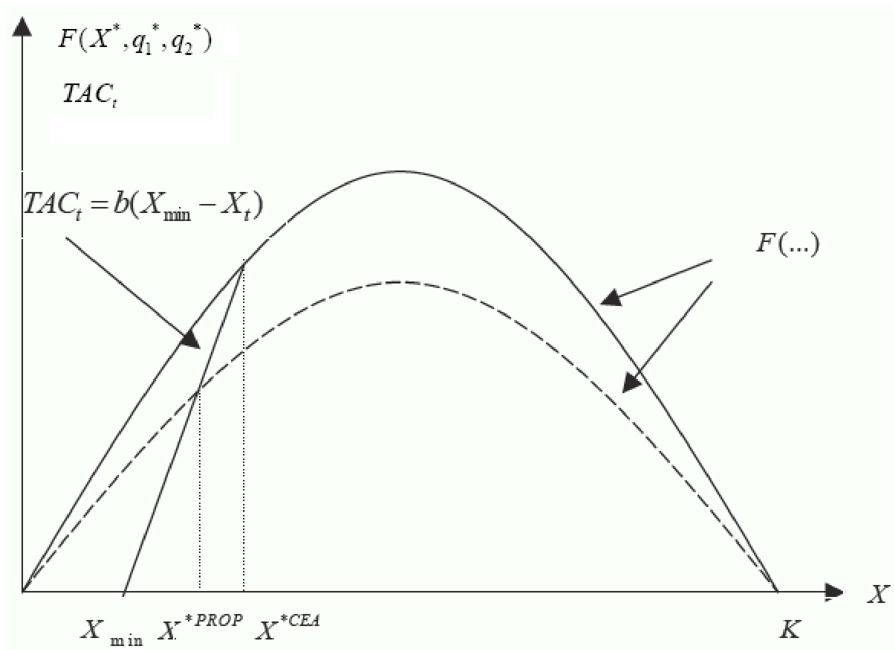

Figure 6.1. Steady-State and Stock Size using CEA and the Proportional Rule

Discussion

This dynamic fishery bankruptcy model with CEA as the proposed outcome is certainly not the only alternative to the proportional rule currently adopted. For instance, one adaptation to the model could be made by usage of the equal awards rule rather than its constrained version. This rule even stronger decreases the influence of unselective fisheries by dividing the TAC equally amongst trawler and coastal fleet, also if this amount would exceed the latter party's claim. In this context the capacity of the coastal fleet should be capable though of meeting the allocation – something that according to Pauly (Pauly & Watson, 2003) should not form any constraint in the present situation.

Furthermore, Gallastegui, Iñarra and Prellezo (2002) compared the results of the standard proportional division rule of the TAC with the nucleolus and Shapley value. Instead of two claimants though, a Bankruptcy Problem was formulated where the TAC had to be divided amongst several nations. Referring back to chapter 4 a short comparison of relevant features will be made at this point. Building from the properties discussed before, the same analysis can be applied here: what is a fair allocation mechanism for the parties involved? First of all sustainability of the seas and fish stocks is of course an important side-constraint to what such an allocation embodies, however these considerations should be already taken into account in the set-up of the TAC amount. Thus, focusing only on the justness for the claiming countries involved, the following list of properties can be regarded as suitable for a fishery Bankruptcy Problem:

- *Claim boundedness:* nations should not be allocated more than they claim. For $i \in N$, $x_i(TAC,d) \le d_i$;
- *Symmetry*: nations that claim the same amounts should also receive the same allocation. For $i,j \in N$, $d_i = d_j \Rightarrow x_i(TAC,d) = x_j(TAC,d)$;
- *Order preservation*: a nation claiming less than another party should not be allocated more than the other nor lose more. For $i,j \in N$, $d_i < d_j \Rightarrow (x_i(TAC,d) \le x_j(TAC,d), d_i - x_i \le d_j - x_j)$;
- *Net worth monotonicity*: under a larger TAC, no party should be allocated less than before. $\Sigma_{i \in N} d_i > TAC' > TAC \Rightarrow x(TAC',d) \ge x(TAC,d)$;
- *Invariance under claims truncation*: a nation with a historical catch that exceeds the (current) TAC and that therefore over-fished should not be allocated more rights based on this past amount. $x(TAC,d) = x(TAC,mTAC)$ where $mTAC_i = \min\{TAC, d_i\}$;
- *Minimal rights*: an allocation should take into consideration a nation's minimum right, i.e. the positive difference between the sum of claim amounts of other nations and the TAC. Thus, minimum right or $m_i = \max\{0, TAC - \Sigma_{j \in N \setminus \{i\}} d_j\}$ and $x(TAC,d) = m(TAC,d) + x(TAC - \Sigma_{i \in N} m_i(TAC,d_i), d - m(TAC,d))$;
- *Non-manipulability* (or *strategy-proofness*): nations should not be able to gain more fishing rights by working together with other nations, or by splitting their claims. For $S \subset N$, $i \in S \Rightarrow x_k(TAC,\tilde{d}) = \sum_{i=k}^{n} x_i(TAC,d)$ where $\tilde{d} = (d_1, d_2, ..., \tilde{d}_k)$ and $\tilde{d}_k = d_k + d_{k+1} + ... + d_n$.

In table 6.1 the properties that are satisfied by the three proposed division rules are shown. Based on only these properties, the nucleolus and Shapley value could both be presented as a more desirable alternative to the proportional rule, given that *non-manipulability* could be fixed by implementing regulations that enforce this specific property. Obviously such adjustments can also be made for the two unsatisfied properties of the proportional rule, however this would change the nature of the rule that as a result would not classify as truly proportional anymore. Noted should be though that the *minimal rights* property is not very likely to apply in a fishery Bankruptcy Problem as the TAC will rather be exceeded by the sum of total claims and therefore no minimum rights would exist. Another relevant property that could be added to the above list is *exemption* or *sustainability*, viz. that the claims of the smaller claimants should be completely covered as if larger claimants would have harvested accordingly, no Bankruptcy Problem would be existent in the first place. However, this property is not satisfied by any of the three rules considered here.

	Proportional	Nucleolus	Shapley
Claim Boundedness	Yes	Yes	Yes
Symmetry	Yes	Yes	Yes
Order Preservation	Yes	Yes	Yes
Monotonicity	Yes	Yes	Yes
Invariance under Claims Truncation	No	Yes	Yes
Minimal Rights	No	Yes	Yes
Non-Manipulability	Yes	No	No

Table 6.1. Overview of Properties for Common TAC-Allocation Rules

6.2 The New Member Problem

Another application of the Bankruptcy Problem can be found in situations where another party plans on joining the already active fishing entities and abide by their rules and regulations, for instance nations within the European Union or (types of) fisheries within a country or region. One of the new measures of the CFP after the reform of 2002 however is to limit new entries to the already existing fleets, e.g. by offering financial benefits to parties that withdraw their fishing activities (European Commission, 2004). Therefore, considered will be only the case of a new nation entering the EU rather than other levels of New Entrant Problems. As already mentioned before, bilateral agreements exist for cases where a sea is shared between both (non)-EU members. Nonetheless, when one of these nations from outside the EU officially becomes a member-state, the existing agreements *will* be reformed and TAC allocations have to be adjusted.

This situation closely resembles the general problem of how to divide the TAC amongst the member states of claimants. In the former application the possible division-rules of CEA, the proportional rule, nucleolus and Shapley value were mentioned as solutions to this issue, all with claims that were based on historical catches. However, is it correct for both entrant and member parties to base their rights on past activities – even more so in this situation than in a case of distribution amongst *only* member-states – or should there be additional criteria to base an allocation on, one that takes into account relevant differences between the established member states and the new country?

Pintassilgo (2000) investigated the New Member Problem in the situation of a Regional Fisheries Management Organization (RFMO). These are management associations supported by the United Nations that cover mostly deep-sea areas where fisheries from various countries can be active. TAC quotas are also used to limit the total harvest in these regions. Under the UN Agreement the current members of a RFMO are not allowed to prevent other parties from entering; new members should be included and be allocated a fair part of the TAC. At the same time though, Pintassilgo argues that entrants form a threat to the stable cooperation of the established RFMO. On short term, if the fish stock and therefore the TAC are small, cooperation can be maintained even if a new member takes away part of the others' share. This is, if the established members choose to disagree and leave the RFMO, their profits will even be smaller due to the fact that any fishery can harvest in the area (i.e., 'tragedy of the commons'). However, on long-term when the recovered fish stock is in a steady-state situation, the new entrant does pose a threat since the profits of the established members will not be as large as without the new member and a non-cooperative situation may seem a better option. Thus, it is also in the interest of environmental sustainability to give entrants incentives to join the existing coalition, since they would rather (over)-fish in another area otherwise. Although the participation of fisheries or nations within a RFMO is not the same as being part of an enforceable treaty such as in the EU, in this context the same procedures could be applied for entrants.

The Waiting Period and Transferable Membership

Two solutions that were put forward by Pintassilgo (2000) to manage incoming parties are the waiting period and a system of transferable rights. Under the former system, entrants are simply excluded from any harvesting rights prior to official entrance with the result that the existing members temporarily benefit from the relatively higher profits. Unfortunately, while the waiting period solution proves to stabilize the RFMO at first, it does not provide a long-term solution to the New Member Problem.

The other solution is one in which the entrant obtains a share of harvesting rights by buying some of the quota from the other parties, treating the TAC allocation of that member as a property right. This method is similar to the Individual Transferable Quota (ITQ) system that has recently been adopted by several fishery management organizations. Transferable quota are seen as an economically efficient solution to the New Member Problem, as entrants will not purchase rights if they do expect their harvest to be less efficient than that of the party trading its rights. On the other hand, parties intending to be active harvesters on long-term basis will purchase a larger share of ITQ. In figure 6.2 Arnason (2007, p.6) depicts the effect an ITQ system has; under efficient operation the optimal situation *e** could be reached. However, as an ITQ turns harvesting rights rather than the actual common property into a property right, such an optimal situation is not guaranteed; the TACs should also be set at the proper level. It is interesting though to note how the development of the ITQ price can indicate whether the TAC is optimal, taken the TAC as a value that can be set within margins instead of being one predetermined amount. Since along with maximizing profit the quota price is optimized as well, an increase (decrease) in the price of ITQs means that the preceding change in the TAC made the TAC level move closer to (further from) the optimal. Here the ITQ price could be interpreted as the marginal value of a unit of fish to the market but also as the cost incurred by the current owner of the quota (Arnason, 2007).

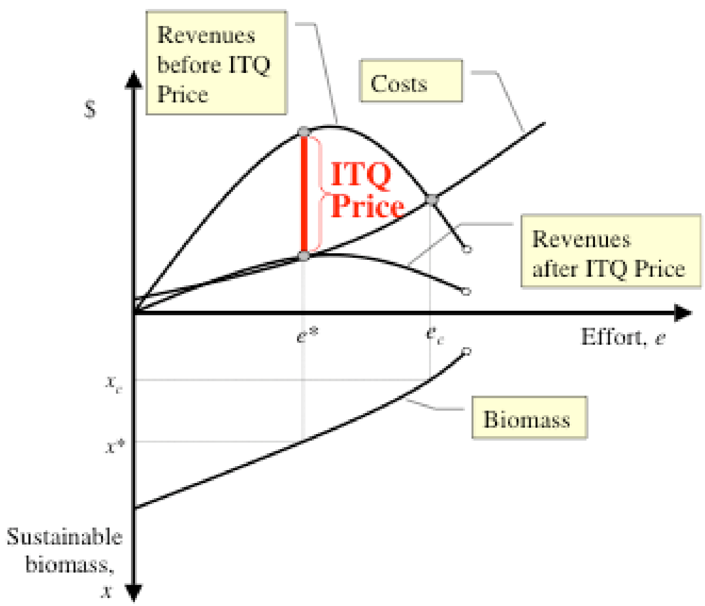

Figure 6.2. The Effect of an ITQ System

However, coming back at the question raised in the beginning of this chapter, it remains unanswered what a just initial allocation scheme is for the ITQ-system to build on (Pintassilgo, 2000). A fairly easy determinant could be again the historical catches of members, yet even more strongly than small seas surrounded by certain countries, deep-sea areas are subject to the notion of common property. Some nations may not have harvested in a specific area for decades, yet this should not mean they deserve less fishing rights than other fisheries in an ocean-zone located distantly from both nations' borders. Thus, although the implementation of an ITQ-system could prove to be successful, first of all the basic assumption of how the quotas are established should be dealt with.

A Monotonic Solution

A third solution is proposed to Hang Pham Do, Folmer and Norde (2006) who sought after an allocation mechanism for profits of (regional) fisheries in case of a new entrant by formulating the situation as a cooperative game. The core idea of their strategy is to develop a division scheme of fishing rights on which both the entering party and the existing fleet agree to continue their fishing activities.

Formally, entrant q wants to join coalition S, which results in a problem of how to allocate the TAC to $S \cup \{q\}$ where the new allocation vector is $(x_i, S \cup \{q\})_{i \in S \cup q}$. The property paramount to an optimal solution according to Hang Pham Do et al. (2006) is *monotonicity*, meaning that all players in the game are satisfied with the new situation. The mechanism that produces an allocation vector that is monotonic for all coalitions is called a population monotonic allocation scheme (PMAS) (Sprumont, 1990). Now, $(x_i, S)_{S \subseteq N, i \in S}$ satisfies PMAS if:

1) $\Sigma_{i \in S} x_{i,S} = v(s) \; \forall \; S \subseteq N$, viz. *efficiency*;
2) $x_{i,S} \leq x_{i,T} \; \forall \; S, T \subseteq N$ with $S \subseteq T$ and $\forall \; i \in S$, meaning that the payoff vector could not be smaller under a coalition T that includes more players than the smaller coalition S.

Furthermore, each 'fishery' Bankruptcy Game is convex, implying that it has a non-empty core. In such a game the players or members could gain from a situation where a new party joins the coalition, as for every S,T such that $S \subseteq T$, $v(S) - \Sigma_{q \in S} v(\{q\}) \leq v(T) - \Sigma_{q \in T} v(\{q\})$. Also, as it is shown in chapter 4, the Shapley value embodies the properties of *efficiency* and *individual rationality*, which makes this an applicable solution method. In fact, the Shapley value of a Bankruptcy Game can be found as the midpoint of the core; in other words, coalitions are enlarged by equal amounts of the value gained with the merge. This 'net gain value' of coalition S and entrant $\{q\}$ is therefore $ngv(S, \{q\}) = v(S \cup \{q\}) - v(S) - v(\{q\}))$. Since this value is nonnegative for disjoint S and $\{q\}$ (i.e. the property of *superadditivity*) the entrance of a new member to the coalition does not destabilize the original group (Hang Pham Do et al., 2001). Thus, it is shown that under an equal division of the net gain value each player is better off choosing for the enlarged coalition than for the initial group to disintegrate. Although the parties of the established coalition will not gain in absolute terms, they are better off taking the entrant in than in a situation where all fisheries operate individually; theoretically no reason is present to make member nations deviate from this situation.

Discussion

Before ending this chapter, once again a brief discussion will be conducted regarding an appropriate basis on which to allocate fishing rights. Finding a suitable and just allocation mechanism to assign quota to member states within the European Union TAC system is already complex, as seen in chapter 6.1, but even more so when this situation is combined with new coalitions of countries. There are some aspects though inherent to the EU management system that do make the situation somewhat easier. Since, contrary to the choice of nations to join a RFMO, it is assumed that the nation planning on joining the EU will not refrain from doing so based on only its future fishing-rights, it follows that there is no need for the entrant to have its conditions improved. Simultaneously, if the new member state neglects her new obligations under the CFP, punishment will cease this behavior.

However, under the assumption that the European Union will strive after an allocation that is the best for its member states, ideally a large scale of relevant aspects do need to be taken into consideration when a new TAC division is made. To provide several examples, first of all the *economic dependence* of a nation on the fishery industry should be sustained. As it can be said that especially new entrants to the EU are susceptible to sudden changes in such an influential economical area, no transformations should be made that lead to imbalance. Moreover, a downfall in economical stability will harm the Union overall. Furthermore, the status quo in *export and import* of fish should be regarded, where it is important to bolster comparative advantages or efficiencies of nations in the harvest of specific fish-types in specific areas.

Following the dynamic bankruptcy model of chapter 6.1, also the *equipment* used by nations should be a determinant of their fishing rights, as states adopting harmful fishing techniques could be punished by being allotted a smaller share. Similarly the level of *sustainable management* and serious efforts to pursue the CPF agreements could be rewarded c.q. punished.

From the perspective of the consumer, *dietary patterns* could also be incorporated on grounds of proper health care. Considering fish as an elementary component of nutrition, all human beings should be able to obtain a minimum amount of fish. However, based on historical catch a country with a (historically) larger consumption could therefore claim to need relatively many fishing rights, with that reducing the harvesting rights of countries having had relatively little in the past but needing more at present. Contrary, nations with a clearly increasing trend in consumption could claim its population is in need of a larger quota as the other nations have been ‘over-consuming’. In the context of the European Union it can be assumed though that no such extreme differences apply.

Furthermore, also *cultivated fish* and fish caught in *inland waters* should be counted towards the total supply of fish, although the former catch does not contribute to the TAC and the latter type of harvest does not need sharing with other EU member states. It could be reasoned that a country with significant inland supply of fish or large (sustainable) breeding facilities has a smaller claim on fish harvest elsewhere.

Finally, an interesting consideration is the *population size* and especially *density* in a certain area of the country. For instance, it could be considered unfair to allocate a considerably large proportion of the TAC to the Russian Federation (in a bilateral agreement with this non-member state) on the basis of population size since only relatively few people indeed live at the border of the Baltic Sea. Instead, some ratio between *coast-line* and population size could be suggested, where the population size could also be substituted by the population density over several areas within the country. Putting a higher weight on the population near the sea border in order to divide the TAC could therefore be perceived as more fair and moreover, this could imply that the harvested fish is indeed consumed in the near area, which is more environmentally friendly as it requires less *transportation*. Naturally, a large country with a small borderline with the sea would also be more dependent on import if it is not allocated a relatively large portion of the TAC, which brings us back again at the import/export consideration.

As it can be seen, a complex system of parameters and weights should be incorporated in order to arrive at an optimally fair allocation scheme, where it should be noted that interdependence of the aspects mentioned is not even fully taken into consideration and thus many correlations are present. One thing is for certain though: a TAC-claim based on historical catches only and combined with a proportional allocation rule is not a fair construction taking all the above into account. Putting the EU in the position of ‘master allocator’, an economic construction that most efficiently distributes the fish needed based on amongst other factors import/export parameters and shortest transportation routes could eventually be of benefit to both nature and human beings. However complicated the implementation of such an optimal system may be, a midway should at least be considered in order to improve the existing management system.

Chapter 7

Conclusion

The global fishing industry is in need of a proper management system to make it – especially when looking at the current situation – refrain from falling into inefficiency, over-fishing and causing harm to the ecological balance, where it should be noted that the notion of fish being common property is a major determinant of these issues. In order to keep up with changing trends, the development of the EU Common Fisheries Policy already went through various revisions such as an increased focus on sustainable fish stocks by limiting the days of fishing activity and increasing the number of protected sea areas. These kinds of aspects are clearly of paramount importance to a better ecological situation, however another focus could be on the division system of the TAC amongst nations. Trying to improve this system rather than seeking after a complete new alternative, the bankruptcy matter comes in useful. In this final chapter, a few suggestions are made to conduct further research.

Two situations where the bankruptcy matter has been applied are those of finding an allocation rule that improves both the recovery time of a fish stock and the steady-state situation, and one in which the entrance of a new member to a coalition of fishing parties is analyzed. Both scenarios can be translated into Bankruptcy Problems that differ in the composition of the claimants, namely the coastal fleet vs. deep-ocean trawlers in the former situation and various member states in the latter. As part of a more elaborate research, also different compositions of claiming parties could be considered and a larger set of allocation mechanisms compared, including cooperative game theoretical solutions. All such findings are relevant pieces working towards a reform of the EU fisheries management, yet they cannot be seen separately.

Shortly recalling the findings of the dynamic Bankruptcy Problem, the CEA rule was proposed as a better allocation rule compared to the currently adopted proportional rule, for the reason that it results in a shorter recovery period and better steady-state conditions. Following another argument, the proportional rule unduly rewards fleets that have harvested more (or too much) in the past; now a *moral judgment* is made rather than focusing on improving the situation. Similarly, many different drawbacks could be found to properties of rules and thereby to rules themselves. It is proposed to make such analysis in the context of EU fisheries management, of course realizing that compromises and trade-offs need to be made where it concerns conflicting property-preferences within allocation rules: whereas one rule could be perceived as the best when it comes to a specific property, looking at several different properties at once, another method could prove to be more suitable. Finally, countries' specific conditions – that make up a large and complex pool of considerations – could also lead to a preference of certain rules.

Another aspect that should be covered in further research is what the underlying assumptions are of the situation in focus. For instance, in the solutions of transferable membership or a PMAS in the New Member Problem, there still is the unclosed discussion of what the ITQ or claim should be based on, how these should be constructed. A detailed analysis of allocation rules can only be made if there is a consensus about the main assumptions, i.e. the meaning of elements in the formulated Bankruptcy Problem.

All in all, the discussed bankruptcy matter can be seen as a versatile tool that could be adopted for further research on the sustainability of the fisheries industry. In order to maximize its use, various Bankruptcy Problems should be considered where a large set of allocation rules and their properties should be analyzed, eventually trying to combine all considerations and weighting them in order to come-up with a satisfactory model. Due to the scope such analysis would require considerable efforts, yet the effect a reform of the allocation scheme may have on the situation of both fish stocks and industry could prove to be the more rewarding.

Chapter 6.1

R	'Gross' intrinsic growth rate
K	Carrying capacity
γ_1	Selectivity parameter coastal fleet
γ_2	Selectivity parameter trawler fleet
$X_{\min}$	Threshold level TAC
B	Proportionally factor TAC
h_1	Historical fishing rights coastal fleet
h_2	Historical fishing rights trawlers
p_1	Harvesting price coastal fleet
p_2	Harvesting price trawler fleet
c_1	Harvesting cost coastal fleet
c_2	Harvesting cost trawler fleet
θ_1	Catchability coefficient coastal fleet
θ_2	Catchability coefficient trawler fleet
δ	Discount rent

List of Parameters of the Dynamic Fishery Bankruptcy Model

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