Reasoning by cases using arguments

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Abstract

An argumentational interpretation of defeasible rules will be developed based on 1) the view that a defeasible rule describes an abstract relation between concepts ignoring exceptional situations, and 2) the view that a disjunction describes alternative situations or extensions. The latter enables us to realize *reasoning by cases* in an argumentational interpretation.

Finally, an equivalent default logic is presented.

1 Introduction

Argumentational interpretations originate from philosophy [15]. More recently they have been studied in AI [2, 3, 4, 5, 6, 7, 8, 10, 11, 14, 15, 16, 17]. In an argumentational interpretation, a defeasible rule is viewed as a warrant for believing the consequent of the rule whenever we have a warrant for believing its antecedent [15]. A warrant for believing the antecedent can consist of facts about the world, denoted as evidence or premises, and of propositions that are warranted by other defeasible rules. So, we can construct a tree of defeasible rules that warrants the belief in some proposition with respect to some evidence. This tree is called an *argument* for the proposition.

It is possible to construct arguments for two or more propositions that are in conflict. A number of these conflicts can be resolved by comparing the arguments involved and subsequently decide which arguments are valid. There is, however, no consensus on how to use the arguments for resolving conflicts. Simari and Loui [14] define a specificity ordering between arguments based on Poole's definition of specificity [9]. Here sets of rules are compared to determine the specificity ordering on arguments. Vreeswijk [17] uses a preference relation on arguments to resolve conflicts. As a consequence, cumulativity and left logical equivalence are not valid. Prakken [11] argues that in legal argumentation, conflict between arguments are resolved by comparing the last rules of the arguments. For specificity, however, Prakken makes an exception. Since he failed to find a suitable order on rules that model specificity, he bases his definition of specificity on Poole's definition of specificity [9].

Roos [13] shows that it suffices to consider only the last rules of an argument for an inconsistency. He also defines a corresponding specificity order. In the paper, we use this approach as a starting point and extend it to enable reasoning by cases.

Reasoning by cases is another problematic area of argumentational interpretation. In general, argumentational interpretations do not possess the ability to reason by cases [2, 3, 4, 5, 10, 11, 14, 15, 16, 17]. Pollock's argument system [6] might be a possible exception. However, he does not describe reasoning by cases explicitly.

2 An argumentational interpretation

In an argumentational interpretation, a defeasible rule is viewed as a warrant for believing the consequent of the rule whenever we have a warrant for believing its antecedent [15]. A warrant for believing the antecedent can consist of facts about the world, denoted as evidence or premises, and of propositions that are warranted by other defeasible rules. So, we can construct a tree of defeasible rules that warrants the belief in some proposition with respect to some evidence. This tree is called an *argument* for the proposition.

If a defeasible rule would describe a valid relations between two propositions, there would be no reason for any defeat. We will assume, however, that a rule describes an *abstract* relations between propositions. Such an abstract relation is the result of formulating a generalized relation by extracting common qualities from specific examples. This generalized relation ignores exceptional situations in which the relations is not valid.

Since a defeasible $\varphi \rightsquigarrow \psi$ rules describes a generalized common relation, there are situations in which the relation it represents is invalid. In these exceptional situations, either $\neg\psi$ must hold or both ψ and $\neg\psi$ must be unknown. Following Pollock [6], the former situation is called *rebutting* defeat and the latter is called *undercutting* defeat. Brewka [1] uses the terms *hard* and *weak* exceptions.

In case of rebutting defeat, we can construct an argument for some propositions and for its negation. The question is: 'which argument corresponds with the exception'. To answer this question we must first formally define an argument.

Definition 1 *Let $\langle \Sigma, D \rangle$ be a defeasible theory where Σ is the set of premises and D is the set of rules.*

Then an argument[1] A for a proposition ψ is recursively defined in the following way:

- *For each $\psi \in \Sigma$: $A = \{\langle \varnothing, \psi \rangle\}$ is an argument for ψ.*
- *Let $A_1, ..., A_n$ be arguments for respectively $\varphi_1, ..., \varphi_n$. If $\varphi_1, ..., \varphi_n \vdash \psi$, then $A = A_1 \cup ... \cup A_n$ is an argument for ψ.*
- *For each $\varphi \rightsquigarrow \psi \in D$ if A' is an argument for φ, then $A = \{\langle A', \varphi \rightsquigarrow \psi \rangle\}$ is an argument for ψ.*

Let $A = \{\langle A'_1, \alpha_1 \rangle, ..., \langle A'_n, \alpha_n, \rangle\}$. Then:

$$\vec{A} = \{\alpha_1, ..., \alpha_n\} \cap D;$$
$$\hat{A} = \{c(\alpha_1), ..., c(\alpha_n)\} \text{ where } c(\alpha) = \alpha \text{ if } \alpha \notin D \text{ and } c(\alpha \rightsquigarrow \beta) = \beta;$$
$$\bar{A} = \{\alpha_i \mid 1 \leq i \leq n, \alpha_i \in D\} \cup \bigcup_{i=1}^{n} \bar{A}'_i;$$
$$\tilde{A} = \{\alpha_i \mid 1 \leq i \leq n, \alpha_i \in \Sigma\} \cup \bigcup_{i=1}^{n} \tilde{A}'_i$$

Example Let $A = \{\langle \varnothing, \alpha \rangle, \langle \{\langle \{\langle \varnothing, \beta \rangle\}, \beta \rightsquigarrow \gamma \rangle\}, \gamma \rightsquigarrow \delta \rangle\}$ be an argument. Then $\vec{A} = \{\gamma \rightsquigarrow \delta\}$ denotes the last rules used in the argument A. Furthermore, $\hat{A} = \{\alpha, \delta\}$ denotes the propositions that represent the base belief set of the argument A. The base belief set is equal to $Th(\{\alpha, \delta\})$. Clearly, A is an argument for every proposition in $Th(\{\alpha, \delta\})$. $\bar{A} = \{\gamma \rightsquigarrow \delta, \beta \rightsquigarrow \gamma\}$ denotes the set of all rules in A, and $\tilde{A} = \{\alpha, \beta\}$ denotes the premises used in the argument A. □

Two arguments can be related to each other. The relation that is of interest for us is whether one argument uses the same inference steps as another argument. If so, the former is called a sub-argument of the latter. Though an argument can be viewed as a tree, a sub-argument is not exactly a sub-tree. A sub-argument cannot be a sub-tree because a tree representing an argument consists of two different types of nodes.

[1]We will sometimes add the index ψ to an argument (A_ψ) to denote that it is an argument for ψ. Of course there can be more than one argument for ψ.

Definition 2 *An argument A is a sub-argument of B, $A \leq B$, if and only if every $\langle A', \alpha\rangle \in A$ is a sub-structure of the argument B.*

$\langle A', \alpha\rangle$ is a sub-structure of an argument B if and only if

- *either there exists a $\langle B'\alpha\rangle \in B$ such that A' is a sub-argument of B';*
- *or there exists a $\langle B', \beta\rangle \in B$ such that $\langle A', \alpha\rangle$ is a sub-structure of B'.*

Example let $A = \{\langle\varnothing, \alpha\rangle, \langle\{\langle\{\langle\varnothing, \beta\rangle\}, \beta \rightsquigarrow \gamma\rangle\}, \gamma \rightsquigarrow \delta\rangle\}$ be an argument. Then

$$\{\langle\varnothing, \alpha\rangle, \langle\{\langle\{\langle\varnothing, \beta\rangle\}, \beta \rightsquigarrow \gamma\rangle\}, \gamma \rightsquigarrow \delta\rangle\}$$
$$\{\langle\varnothing, \alpha\rangle, \langle\{\langle\varnothing, \beta\rangle\}, \beta \rightsquigarrow \gamma\rangle\}$$
$$\{\langle\varnothing, \alpha\rangle, \langle\varnothing, \beta\rangle\}$$
$$\{\langle\varnothing, \alpha\rangle\}$$
$$\{\langle\{\langle\{\langle\varnothing, \beta\rangle\}, \beta \rightsquigarrow \gamma\rangle\}, \gamma \rightsquigarrow \delta\rangle\}$$
$$\{\langle\{\langle\varnothing, \beta\rangle\}, \beta \rightsquigarrow \gamma\rangle\}$$
$$\{\langle\varnothing, \beta\rangle\}$$

are sub-arguments of A. □

An argument represents a derivation tree of defeasible rules. Since a rule in an argument A gives a warrant for its consequent, the argument can be viewed as a global warrant for a proposition φ, $\hat{A} \mathrel{|\!\!\sim} \varphi$, that is grounded in the premises $\tilde{A}$. Whether an argument is valid depends on whether the argument or one of its sub-arguments is defeated. When an argument A for some proposition φ is valid we say that φ follows from the premises $\tilde{A}$ using the rules $\bar{A}$.

Conflicts

Arguments supporting propositions that are inconsistent are said to disagree. If $A_\perp = \{\langle A_1, \alpha_1\rangle, ..., \langle A_n, \alpha_n\rangle\}$ is an argument for $\perp$, ($\hat{A}_\perp \mathrel{|\!\!\sim} \perp$), then the arguments $A_1, ..., A_n$ are said to *disagree*. Disagreeing arguments must be compared in order to resolve the derived inconsistency. After comparing the arguments we may decide that one of the arguments is *defeated* by the others. By this we mean that the argument may no longer warrant a proposition because of the other arguments.

What should it exactly mean: an argument is defeated by other arguments? For example, does it mean that only the last rule in the argument may no longer warrant its consequent or can it also mean that one of its proper sub-arguments is also defeated. Let us assume that latter case is allowed for. Suppose that $\{\psi, \mu\}$ is an inconsistent set, that ψ is warranted by the argument

$$A_\psi = \{\langle\{\langle\{\langle\varnothing, \alpha\rangle\}, \alpha \rightsquigarrow \varphi\rangle\}, \varphi \rightsquigarrow \psi\rangle\}$$

and that μ is warranted by

$$A_\mu = \{\langle\{\langle\varnothing, \eta\rangle\}, \eta \rightsquigarrow \mu\rangle\}.$$

If A_μ defeats the sub-argument

$$A_\varphi = \{\langle\{\langle\varnothing, \alpha\rangle\}, \alpha \rightsquigarrow \varphi\rangle\}$$

of A_ψ, then the situation in which α and η hold ($\alpha \wedge \eta$) represents an exception on the rule $\alpha \rightsquigarrow \varphi$. In this exceptional situation either $\neg\varphi$ holds or φ is unknown.

Suppose that $\neg\varphi$ holds. Then there must be a set of defeasible rules $\{\xi_1 \rightsquigarrow \nu_1, ..., \xi_k \rightsquigarrow \nu_k\}$ such that $\xi_1, ..., \xi_k$ are derivable from $\eta \wedge \alpha$ and $\{\nu_1, ..., \nu_k\} \cup \Sigma$ imply $\neg\varphi$. But then, there is no longer a need for allowing that A_μ defeats A_φ since

$$A_{\neg\varphi} = \{\langle A_{\xi_1}, \xi_1 \rightsquigarrow \nu_1\rangle, ..., \langle A_{\xi_k}, \xi_k \rightsquigarrow \nu_k\rangle, \langle\varnothing, \sigma_1\rangle, ..., \langle\varnothing, \sigma_j\rangle\}$$

already defeats A_φ.

If, on the other hand, φ is unknown, the disagreeing arguments A_ψ, A_μ must somehow imply that $\neg(\alpha \rightsquigarrow \varphi)$ holds. The only way to formulate this is by a rule $\xi \rightsquigarrow \neg(\alpha \rightsquigarrow \varphi)$ where ξ is derivable from $\eta \wedge \alpha$. Again, there is no need for allowing that A_μ defeats A_φ since

$$A_{\neg(\alpha\rightsquigarrow\varphi)} = \{\langle A_\xi, \xi \rightsquigarrow \neg(\alpha \rightsquigarrow \varphi)\rangle\}$$

already defeats A_φ.

Hence, no proper sub-argument of the disagreeing arguments is defeated in order to solve the conflict. Only one of the last rules may no longer warrant its conclusion.

> A conflict described by a set of disagreeing arguments must be resolved by defeating the last rule of one of the disagreeing arguments.

How do we determine the which of the last rules of a argument for an inconsistency must be defeated? We will assume that we only have to consider the last rules of the argument to determine the one to be defeated. The advantage of this assumption is that the resolution of inconsistencies is cumulative provided that there are no odd attack loops in the set of arguments. It does not matter whether the antecedent of a last rule is an observed fact or derived through reasoning. Furthermore, an observed fact may be based on some hidden reasoning of which we are not aware.

To determine which rule is defeated by other rules after deriving an inconsistency, usually, some preference relation is used. The most commonly used preference relation is a *specificity* order. In legal argumentation other preference relations are applied as well [11]. These preference relations are specified by meta-rules. This makes them subject to defeat in situations where the meta-rules specify incompatible relations between rules.

Definition 3 *Let $\langle\Sigma, D, \succ\rangle$ be a defeasible theory where $\succ$ is a partial preference relation on D. Furthermore, let*

$$A_1 = \{\langle A'_1, \eta_1 \rightsquigarrow \psi_1\rangle\}, ..., A_k = \{\langle A'_k, \eta_k \rightsquigarrow \psi_k\rangle\},$$
$$A_{k+1} = \{\langle\varnothing, \sigma_1\rangle\}, \ldots, A_n = \{\langle\varnothing, \sigma_j\rangle\}$$

be disagreeing arguments.

Then, if $\eta_i \rightsquigarrow \psi_i$ is the least preferred last rule in $\vec{A}$ where $A = A_1 \cup ... \cup A_n$, $\eta_i \rightsquigarrow \psi_i$ is defeated.

Reasoning by cases

Argumentational interpretations lack, in general, the ability of reasoning by cases. Through reasoning by cases, one can derive new conclusions as is illustrated by the following example. Suppose that we have two rules, one stating that someone has a handicap if this person's left arm is broken and one stating that someone has a handicap if this person's right arm is broken. Then, knowing that John has broken one of his arms, we should be able to conclude that John has a handicap. Unfortunately, no argument can be constructed for this line of reasoning.

To make things even more complicated, rules can be defeated through reasoning by cases. In the above mentioned example the rule 'normally, someone does not have a handicap', should be defeated because it is defeated when reasoning by cases. We only consider exceptional situations with respect to this rule.

A rule is defeated if it is defeated in every case when reasoning by cases. Should a rule also be defeated if it is defeated in only one case? For example, may we conclude that Tweety can fly if we know that it is either a penguin or a eagle. The view taken here is that cases must be considered separately whenever they are derivable as a disjunction which is not implied by some other proposition. Therefore, as long as we do not know what kind of a bird Tweety can be, 'Tweety can fly' is derivable. If, however, we know that Tweety is either a penguin or a eagle, the two cases 'Tweety is a penguin' and 'Tweety is a eagle' must be considered separately. Hence, we may not conclude that Tweety can fly because the rule 'birds can fly' is defeated in the case that Tweety is a penguin.

The following example illustrates the reason for evaluating the situations described by a disjunction separately even better. Suppose that we have the following rules:

- If someone sees a person committing a crime, then that person is guilty of the crime.
- If someone sees a person not at the crime scene but elsewhere, then that person has an alibi for the crime.
- A person guilty of a crime must be punished.

Now suppose that Peter sees John robbing a bank and Fred sees John or his brother Paul drinking a beer in a bar at the time of the bank robbery. If we would not evaluate the situations described by the disjunction separately, we will conclude that John is guilty of robbing the bank and therefore that John must be punished. This would be most unfortunate if John would actually be the person that was seen by Fred drinking a beer in the bar. The wrong person would be punished.

Considering cases separately has important consequences for the semantics of the propositions. To avoid the consideration of irrelevant cases, such as $\eta \vee \neg\eta$, a three valued semantics is needed. Without a three valued semantics, we cannot explicitly state that a proposition is unknown. Since a tautology is always true in a two valued semantics, we can defeat any rule for which there is an exception. Consider, for example, the defeasible theory $\langle \Sigma, D \rangle$ where $\Sigma = \{bird\}$ and $D = \{bird \leadsto can\_fly, penguin \leadsto \neg can\_fly\}$. can_fly is not a theorem of this defeasible theory because of the tautology $penguin \vee \neg penguin$. In the case described by $penguin$, the rule $bird \leadsto can\_fly$ will be defeated.

A proposition can be given a truth value based on Kleene's strong three valued semantics. This semantics assigns the value *true* to a proposition $bird \vee penguin$ if $bird$ is *true* and $penguin$ is *unknown*. Since $penguin$ is unknown, we should not consider it when reasoning by cases. Otherwise, again we can defeat any rule for which there is an exception. In the case described by $penguin$, the rule $bird \leadsto can\_fly$ will again be defeated.

The problem can be avoided by assuming that $\alpha \vee \beta$ denotes that there are three possible situations; $\alpha \wedge \beta$, $\neg\alpha \wedge \beta$ or $\alpha \wedge \neg\beta$. If β is unknown, none of these cases can be true. Since, in Kleene's *weak* three valued semantics, a proposition has a known truth value if and only if both sub-propositions have a known truth value, we will use this semantics instead. The semantics is similar to Bochvar's semantics where *unknown* is interpreted as *meaninglessness*.

$\wedge$	t	u	f
t	t	u	f
u	u	u	u
f	f	u	f

$\vee$	t	u	f
t	t	u	t
u	u	u	u
f	t	u	f

A disjunction is viewed as describing three situations. Since a rule can be defeated in one of these situations, we should consider each of these situations separately. This is possible by introducing a special set of rules; the hypotheses. Such a hypothesis should introduce one of the situations described by a disjunction. Let L be the set of all propositional formulas. Then the set of hypotheses H is defined as:

$$H = \bigcup_{\alpha \vee \beta \in L} \{\alpha \vee \beta \leadsto \alpha \wedge \beta, \alpha \vee \beta \leadsto \neg\alpha \wedge \beta, \alpha \vee \beta \leadsto \alpha \wedge \neg\beta\}$$

No preference may be specified among these rules, but every rule in D must preferred to every rule in H; $\forall \varphi \leadsto \psi \in D, \forall \alpha \leadsto \beta \in H : \varphi \leadsto \psi \succ \alpha \leadsto \beta$. Since conflicts between two hypotheses can only be resolved by defeating one of the hypotheses, disjunctions can result in multiple *extensions*.

The belief set

An inconsistency can be resolved considering the last rules of the argument for the inconsistency. This implies that in case the inconsistency is resolved, one of these last rules may no longer warrant the belief in its consequent; i.e. the rule is defeated. For this rule we can construct an argument supporting the defeat of this rule.

Definition 4 *Let $A_\perp$ be an argument for an inconsistency and let $\varphi \leadsto \psi \in min_\succ(\vec{A}_\perp)$ be a least preferred last rule for the inconsistency.*

If $\langle A_\varphi, \varphi \leadsto \psi \rangle \in A_\perp$, then $A_{\neg(\varphi \leadsto \psi)} = (A - \{\langle A_\varphi, \varphi \leadsto \psi \rangle\}) \cup A_\varphi$ is an argument for the defeat of $\varphi \leadsto \psi$.

Given these arguments for the defeat of rules, we can define an *extension*. Here an extension is a set of propositions for which we have valid arguments. A valid argument is an argument of which the rules are not defeated. This also holds for the arguments for the defeat of rules. A rule is defeated if the argument for its defeat is valid; i.e. the argument does not contain defeated rules.

Definition 5 *Let $\mathcal{A}$ be a set of all possible arguments and let $Defeat(\Gamma) = \{\alpha \leadsto \beta \mid A_{\neg(\alpha \leadsto \beta)} \in \mathcal{A}, \bar{A}_{\neg(\alpha \leadsto \beta)} \cap \Gamma = \varnothing\}$.*

Then the set of defeated rules Ω is defined as:

$$\Omega = Defeat(\Omega).$$

Proposition 1 *The set of defeated rules Ω is well founded. I.e. for each $\Lambda \neq \Omega$ such that $\Lambda = Defeat(\Lambda)$, neither $\Lambda \subset \Omega$ nor $\Lambda \supset \Omega$ holds.*

Notice that the set of defeated rules need not be unique. Even if no hypotheses are used and every inconsistency has a unique least preferred last rule, the set of defeated rules need not be unique. Consider for example the facts α and β and rules $\alpha \leadsto \gamma$, $\beta \leadsto \delta$, $\gamma \leadsto \neg\delta$ and $\delta \leadsto \gamma$, where the last two rules are preferred to the first two. Here there are two sets of defeated rules Ω; $\{\alpha \leadsto \gamma\}$ and $\{\beta \leadsto \delta\}$.

Given a set of defeated rules, the extensions and the belief set can be defined.

Definition 6 *Let Ω be a set of defeated rules and let $\mathcal{A}$ be a set of all derivable arguments.*

Then an extension $\mathcal{E}$ is defined as:

$$\mathcal{E} = \{\varphi \mid A_\varphi \in \mathcal{A}, \bar{A}_\varphi \cap \Omega = \varnothing\}.$$

Definition 7 *Let $\langle \Sigma, D, \succ \rangle$ be a defeasible theory. Furthermore, let $\mathcal{E}_1, ..., \mathcal{E}_n$ be the corresponding extensions.*

Then the belief set B is defined as:

$$B = \bigcap_{i=1}^{n} \mathcal{E}_i.$$

3 An extensional interpretation

In the previous section we have seen that conflicts can be resolved by comparing the last rules of the argument for the conflict. This property makes it possible to define an equivalent default logic [12]. Since we only need to consider the last rules, it suffices to verify whether these last rules are applicable –*there antecedents hold*–, and whether they are not defeated by other rules –*there consequents hold*–.

The consequences of a set of applicable rules, together with the premises, may form an inconsistent set of propositions. Since defeasible rules are viewed as normal default rules, one of these rules must be defeated. The partial preference relations on the rules will be used to determine the rule that must be defeated. If an applicable rule is defeated, there must be a set of non-defeated applicable rules that implies, together with the premises, the negation of its consequent. Furthermore, the defeated rule may not be preferred to any of rules that cause its defeat.

Like in argumentational interpretations, in default logic, reasoning by cases is a problem. In the new argumentational interpretation presented in the previous section, this problem has been solved by using Kleene's weak three valued semantics and by viewing disjunctions as describing different extensions. The same approach can be used here. By doing so, we get a simple and elegant way of handling reasoning by cases. Again, not only new propositions can be derived through reasoning by cases, but also propositions can become invalid through reasoning by cases. Therefore, for the same reason as mentioned in the previous section, we need to use Kleene's weak three valued semantics to avoid the defeat of every rule for which an exception exists. Since in Kleene's weak three valued semantics a proposition's truth is known if and only if the truth value of its constitutive parts are known, a disjunction describes possible extensions. Such an extension must be equal to the deductive closure of the set of literals that it contains.

Definition 8 *Let $\langle \Sigma, D, \succ \rangle$ be a defeasible theory.*

Then an extension $\mathcal{E}$ is the inclusion minimal set for which there holds:

1. $\Sigma \subseteq \mathcal{E}$;
2. *$\mathcal{E}$ is equal to the deductive closure of the set of literals it contains;*
3. *if there is a $\Delta \subset D$ that defeats $\varphi \rightsquigarrow \psi$ with respect to $\succ$, then $\neg(\varphi \rightsquigarrow \psi) \in \mathcal{E}$;*
4. *if $\varphi \in \mathcal{E}$, $\varphi \rightsquigarrow \psi \in D$ and $\neg(\varphi \rightsquigarrow \psi) \notin \mathcal{E}$, then $\psi \in \mathcal{E}$.*

Δ defeats $\varphi \rightsquigarrow \psi$ with respect to $\succ$ if and only if

- $\varphi \in \mathcal{E}$,
- $\Delta \subseteq \{\eta \rightsquigarrow \mu \in D \mid \{\eta, \mu\} \subseteq \mathcal{E}\}$,
- $\{\mu \mid \eta \rightsquigarrow \mu \in \Delta\} \cup \Sigma \mid\!-_3 \neg\psi$ *and*
- *for no $\eta \rightsquigarrow \mu \in \Delta$ there holds: $\varphi \rightsquigarrow \psi \succ \eta \rightsquigarrow \mu$.*

Theorem 1 *Let $\langle \Sigma, D \rangle$ be a defeasible theory and E the evidence and let $\succ$ be a preference relation on D.*

Then, the set of extensions determined by the argumentational interpretation is equal to the set of extensions determined by the extensional interpretation if we ignore the defeated hypotheses in the extensions of the former.

Example Let $\langle \Sigma, D, \succ \rangle$ be a defeasible theory where $\Sigma = \{\alpha, \beta \veebar \gamma\}$, $D = \{\alpha \leadsto \delta, \beta \leadsto \neg\delta\}$ and $\succ = \{(\beta \leadsto \neg\delta, \alpha \leadsto \delta)\}$.

Then we can construct the following arguments.

$$
\begin{aligned}
&A_{\beta\wedge\neg\gamma} = \{\langle \{\langle \varnothing, \beta \veebar \gamma\rangle\}, \beta \vee \gamma \leadsto \beta \wedge \neg\gamma \rangle\};\\
&A_{\neg\beta\wedge\gamma} = \{\langle \{\langle \varnothing, \beta \veebar \gamma\rangle\}, \beta \vee \gamma \leadsto \neg\beta \wedge \gamma \rangle\};\\
&A_{\beta\wedge\gamma} = \{\langle \{\langle \varnothing, \beta \veebar \gamma\rangle\}, \beta \vee \gamma \leadsto \beta \wedge \gamma \rangle\};\\
&A_{\delta} = \{\langle \{\langle \varnothing, \alpha\rangle\}, \alpha \leadsto \delta \rangle\};\\
&A_{\neg\delta} = \{\langle \{\langle \{\langle \varnothing, \beta \veebar \gamma\rangle\}, \beta \vee \gamma \leadsto \beta \wedge \neg\gamma \rangle\}, \beta \leadsto \neg\delta \rangle\};\\
&A_{\neg(\beta\vee\gamma\leadsto\beta\wedge\neg\gamma)} = \{\{\langle \{\langle \varnothing, \beta \veebar \gamma\rangle\}, \beta \vee \gamma \leadsto \neg\beta \wedge \gamma \rangle, \langle \varnothing, \beta \veebar \gamma\rangle\}\};\\
&A_{\neg(\beta\vee\gamma\leadsto\neg\beta\wedge\gamma)} = \{\{\langle \{\langle \varnothing, \beta \veebar \gamma\rangle\}, \beta \vee \gamma \leadsto \beta \wedge \neg\gamma \rangle, \langle \varnothing, \beta \veebar \gamma\rangle\}\};\\
&A_{\neg(\beta\vee\gamma\leadsto\beta\wedge\gamma)} = \{\langle \varnothing, \beta \veebar \gamma\rangle\}\};\\
&A_{\neg(\alpha\leadsto\delta)} = \{\langle \{\langle \{\langle \varnothing, \beta \veebar \gamma\rangle\}, \beta \vee \gamma \leadsto \beta \wedge \neg\gamma \rangle\}, \beta \leadsto \neg\delta \rangle, \langle \varnothing, \alpha\rangle\}
\end{aligned}
$$

This set of arguments result in two fixed points, $\Omega_1 = \{\beta\vee\gamma \leadsto \neg\beta\wedge\gamma, \beta\vee\gamma \leadsto \beta\wedge\gamma, \alpha \leadsto \delta\}$ and $\Omega_2 = \{\beta \vee \gamma \leadsto \beta \wedge \neg\gamma, \beta \vee \gamma \leadsto \beta \wedge \gamma\}$, for the function *Defeat*. So, we have two extensions

$$\mathcal{E}_1 = Th_3(\{\alpha, \beta, \neg\gamma, \neg\delta, \neg(\beta \vee \gamma \leadsto \neg\beta \wedge \gamma), \neg(\beta \vee \gamma \leadsto \beta \wedge \gamma), \neg(\alpha \leadsto \delta)\})$$

and

$$\mathcal{E}_2 = Th_3(\{\alpha, \neg\beta, \gamma, \delta, \neg(\beta \vee \gamma \leadsto \beta \wedge \neg\gamma), \neg(\beta \vee \gamma \leadsto \beta \wedge \gamma)\}).$$

According to the extensional interpretation given in this section, an extension must at least contain the premises $\Sigma = \{\alpha, \beta \veebar \gamma\}$. Since an extension must be equal to the deductive closure of the literals that it contains, at least two extensions are possible given the premises; one containing $\alpha, \beta, \neg\gamma$ and one containing $\alpha, \neg\beta, \gamma$.

- Suppose now that in the former, we cannot defeat $\alpha \leadsto \delta$. Then δ must belong to the extension. Furthermore, since $\beta \leadsto \neg\delta \succ \alpha \leadsto \delta$, $\beta \leadsto \neg\delta$ will not be defeated either. Therefore, $\neg\delta$ will belong to the extension. But then $\alpha \leadsto \delta$ will be defeated. Contradiction.

  Hence, $\alpha \leadsto \delta$ must be defeated. Since we cannot defeat $\beta \leadsto \neg\delta$, $\neg\delta$ will belong to the extension. Therefore we can derive $\neg(\alpha \leadsto \delta)$.

- In the latter, we cannot defeat $\alpha \leadsto \delta$. Therefore, δ must belong to the extension.

So, we have two extensions

$$\mathcal{E}'_1 = Th_3(\{\alpha, \beta, \neg\gamma, \neg\delta, \neg(\alpha \leadsto \delta)\})$$

and

$$\mathcal{E}'_2 = Th_3(\{\alpha, \neg\beta, \gamma, \delta\}).$$

Clearly, if we ignore the hypotheses, we have the same extensions in both the argumentational and the extensional interpretation of the defeasible rules. $\square$

The above presented default logic can be reformulated in terms of Reiter's default logic [12]. Firstly, we must translate the defeasible rules into default rules. Since we must be able to denote that the application of a default rule is no longer valid, $\neg(\alpha \rightsquigarrow \beta)$, we will associate a name with each default rule. This name will be used to denote that the rule may no longer be applied. So if $n_{\alpha \rightsquigarrow \beta}$ is the name of the translation of $\alpha \rightsquigarrow \beta$, then $\neg n_{\alpha \rightsquigarrow \beta}$ will be the translation of $\neg(\alpha \rightsquigarrow \beta)$. To ensure that the default rule named $n_{\alpha \rightsquigarrow \beta}$ will not be applied if $\neg n_{\alpha \rightsquigarrow \beta}$ holds, we must use the name of the default rule as one of the justifications of the default rule. Hence, we translate a defeasible rule $\alpha \rightsquigarrow \beta$ into the default rule $\dfrac{\alpha : \beta, n_{\alpha \rightsquigarrow \beta}}{\beta}$.

We must also make a modification in Reiter's definition of a default extension. The modification that we need to make, is changing the deductive closure operator Th. We must replace it by a deductive closure operator based on the weak three valued derivability relation.

$$Th_3(S) = \{\varphi \mid S \vdash_3 \varphi\}$$

This results in the following definition of a *modified Reiter-extension.*

Definition 9 *Let (R, Σ) be a default theory.*

Then, an extension E is defined as a fixed point of the operator Γ; i.e. $E = \Gamma(E)$. $\Gamma(S)$ is defined as the inclusion minimal set that satisfies the following three conditions.

1. $\Sigma \subseteq \Gamma(S)$;
2. $\Gamma(S) = Th_3(\Gamma(S))$;
3. *if* $\dfrac{\alpha : \beta_1, ..., \beta_m}{\gamma} \in D$, $\alpha \in \Gamma(S)$ *and* $\neg\beta_1, ..., \neg\beta_m \notin S$, *then* $\gamma \in \Gamma(S)$.

The above defined Reiter-extension need not be equal to the deductive closure of the literals it contains. In order to guarantee this, we can use the same approach as we have used for the argumentational interpretation; i.e. using a set of hypotheses.

$$Hd = \bigcup_{\alpha \vee \beta \in L} = \{\frac{\alpha \vee \beta : \neg\alpha \wedge \beta}{\neg\alpha \wedge \beta}, \frac{\alpha \vee \beta : \alpha \wedge \beta}{\alpha \wedge \beta}, \frac{\alpha \vee \beta : \alpha \wedge \neg\beta}{\alpha \wedge \neg\beta}\}$$

By adding this set Hd to the set of default rules R, we guarantee that an extension is equal to the deductive closure of the set of literals it contains.

The translation of the defeasible rules are all semi-normal default rules. Therefore, there will always exist an extension. It is not difficult to verify that every extension according to Definition 8 is also a Reiter-extension according to Definition 9. Since the preferences relation on the default rules is not taken into account, some extensions according to Definition 9 need not be extensions according to Definition 8. To eliminate these extensions, we must verify for each extension whether it is compatible with the preference relation on the rules. An extension is compatible with the preference relation if for each blocked[2] default rule $\dfrac{\alpha : \beta_1, \ldots, \beta_m}{\gamma}$ there is a set of active[3] default rules $\Delta \subseteq R$ such that the consequences of the rules in Δ together with the premises Σ imply $\neg\beta$, and for no rule $\dfrac{\varphi : \eta_1, \ldots, \eta_n}{\psi}$ in Δ $\dfrac{\alpha : \beta_1, \ldots, \beta_m}{\gamma} \succ \dfrac{\varphi : \eta_1, \ldots, \eta_n}{\psi}$ holds.

[2]The application of a default rule is *blocked* in an extension if its prerequisite belongs to the extension and one of its justifications is inconsistent with the extension.

[3]A default rule is *active* in an extension if its prerequisite belongs to the extension and its justifications are consistent with the extension.

4 Conclusion

In this paper a new argumentational interpretation of defeasible rules has been proposed. The argumentational interpretation has been developed based on 1) the view that a defeasible rule describes an abstract relation between concepts ignoring exceptional situations in which the rule should not be applicable, and 2) the view that a disjunction describes alternative situations or extensions. The non-standard interpretation of a disjunction solves a problem that arises in many argumentational interpretations. It enables reasoning by cases through the construction of an argument for each alternative described by a disjunction.

A default logic, equivalent to the argumentational interpretation of defeasible rules, has been presented. The default logic has been reformulated in terms of Reiter's default logic. This reformulation shows how to enable reasoning by cases in Reiter's default logic.

References

[1] G. Brewka, Cumulative default logic: in defense of nonmonotonic inference rules, *Artificial Intelligence* **50** (1991) 183-205.

[2] C. Cayrol, On the relation between argumentation and non-monotonic coherence-based entailment, *IJCAI-95* (1995) 1443-1448.

[3] P. M. Dung, On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, *IJCAI-93* (1993) 852-857.

[4] A. Hunter, Defeasible reasoning with structured information, *KR'94* (1994) 281-292.

[5] F. Lin, Y. Shoham, Argument Systems, *KR'89* (1989) 245-255.

[6] Pollock, J. L., Defeasible reasoning, *Cognitive Science* **11** (1987) 481-518.

[7] Pollock, J. L., How to reason defeasibly, *Artificial Intelligence* **57** (1992) 1-42.

[8] Pollock, J. L., Justification and defeat, *Artificial Intelligence* **67** (1994) 377-407.

[9] Poole, D., On the comparison of theories: preferring the most specific explanation, *IJCAI-85* (1985) 144-147.

[10] Poole, D., A logical framework for default reasoning, *Artificial Intelligence* **36** (1988) 27-47.

[11] Prakken, H., Logical tools for modelling legal argument, Ph.D. Thesis, Vrije Universiteit, Amsterdam, The Netherlands (1993).

[12] R. Reiter, A logic for default reasoning, *Artificial Intelligence* **13** (1980) 81-132.

[13] Roos, N., Reasoning with defeasible rules, *Dutch/German workshop on non-monotonic reasoning techniques and their application* (1995) 176-183.

[14] Simari, G. R., Loui, R. P., A mathematical treatment of defeasible reasoning and its implementation, *Artificial Intelligence* **53** (1992) 125-157.

[15] Toulmin, S., *The uses of argument*, Cambridge University Press (1958).

[16] Vreeswijk, G., The feasibility of defeat in defeasible reasoning, *KR-91* (1991)

[17] Vreeswijk, G., Nonmonotonicity and partiality in defeasible argumentation, in: Hoek, W. van der, Meyer, J.-J. Ch., Tan Y. H., Witteveen, C. (eds), *Non-monotonic reasoning and partial semantics*, Ellis Horwood (1992) 107-155.